Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 301 (2016) 336–358
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

A morphing approach to couple state-based peridynamics with classical continuum mechanics

Fei Han[a,*], Gilles Lubineau[a,*], Yan Azdoud[a], Abe Askari[b]

[a] *King Abdullah University of Science and Technology (KAUST), Physical Science and Engineering Division, COHMAS Laboratory, Thuwal 23955-6900, Saudi Arabia*
[b] *Propulsion System Engineering, Boeing Commercial Airplane, WA, USA*

Received 26 August 2015; received in revised form 20 December 2015; accepted 22 December 2015
Available online 4 January 2016

Abstract

A local/nonlocal coupling technique called the morphing method is developed to couple classical continuum mechanics with state-based peridynamics. State-based peridynamics, which enables the description of cracks that appear and propagate spontaneously, is applied to the key domain of a structure, where damage and fracture are considered to have non-negligible effects. In the rest of the structure, classical continuum mechanics is used to reduce computational costs and to simultaneously satisfy solution accuracy and boundary conditions. Both models are glued by the proposed morphing method in the transition region. The morphing method creates a balance between the stiffness tensors of classical continuum mechanics and the weighted coefficients of state-based peridynamics through the equivalent energy density of both models. Linearization of state-based peridynamics is derived by Taylor approximations based on vector operations. The discrete formulation of coupled models is also described. Two-dimensional numerical examples illustrate the validity and accuracy of the proposed technique. It is shown that the morphing method, originally developed for bond-based peridynamics, can be successfully extended to state-based peridynamics through the original developments presented here.

Keywords: Morphing method; State-based peridynamics; Classical continuum mechanics; Coupling method

1. Introduction

Recently, Silling [1] developed peridynamics, a nonlocal theory of solid mechanics, based on integral equilibrium equations instead of on the classical use of the partial differential equation. It is assumed that the equilibrium of a material point is attained by an integral of internal forces exerted by nonadjacent points across a finite distance. This nonlocal model is mathematically compatible with crack initiation and propagation, as integral equations can naturally handle discontinuities. These advantages have attracted considerable attention to peridynamics in recent years [2,3].

Peridynamics has been successfully applied to crack propagation [4,5], investigating impact on a brittle solid [6], failure analyses of composites [7,8] and nanotube reinforced composites [9]. The first peridynamic formulation that

* Corresponding authors. Tel.: +966 128082983.
E-mail addresses: fei.han@kaust.edu.sa (F. Han), gilles.lubineau@kaust.edu.sa (G. Lubineau).

http://dx.doi.org/10.1016/j.cma.2015.12.024
0045-7825/

most researchers have applied from the literature is the "bond-based" model. A bond represents interaction forces between pair-wise points. In the bond-based peridynamic (BPD) model, the forces within each bond are central forces that are determined independently of each other. As a result, this model is restricted to the specific constitutive behavior of isotropic materials with a Poisson ratio of $1/3$ in 2 dimensions or $1/4$ in 3 dimensions [1,10].

To break the restriction, Silling et al. reformulated the peridynamic theory called the "state-based" model, in which bond forces remain. However, each bond force is determined depending on the collective deformation of all the bonds in the neighborhood of each endpoint. Thus, the state-based peridynamic (SPD) model overcomes the limitation of Poisson ratios in the BPD model. The SPD model has been employed to study plasticity [11,12], damage and fracture [13,14] of materials.

However, the peridynamic theory, in particular the SPD model comes with an important computational consumption that limits its application. Additionally, peridynamics is characterized by volume-like boundary conditions, rather than the conventional traction-like boundary conditions, increasing the inconvenience of application for engineers. As a result, a reasonable strategy is to preserve peridynamics for "fine-scale" descriptions where key mechanisms are considered to strongly impact the solution such as damage or fracture, and to use continuum mechanics for the other parts of the structure, where the conventional continuum model saves considerable computational costs and satisfies boundary conditions on the premise of solution accuracy. In this way, researchers can couple the SPD and classical continuum mechanics (CCM) models efficiently.

Recently, some coupling schemes have been proposed to combine CCM and BPD models; for example, the variable horizon method [15,16], the force-based coupling method [17,18], the Arlequin coupling method [19] and the morphing method [20]. The variable horizon method blends the local and nonlocal equations by reduction of the peridynamic horizon in the vicinity of the nonlocal model interface where the mathematical incompatibility is greatly reduced. The force-based coupling method blends BPD and CCM models into a coupled force equilibrium equation by a weighting function and a Taylor approximation. On the other hand, the Arlequin coupling method employs a partition of unity approach to couple energy equations of both models in an overlapping domain. Furthermore, the morphing method constructs a balanced relationship between stiffness tensors of the CCM model and the weighted nonlocal parameters of the BPD model through the equivalent energy density of both models. As a result, the morphing method implements the transition between both models by means of only this simple and unified balance over the whole structure, making it a versatile and powerful technique. Some developments in morphing-based coupling between BPD and CCM models have already been achieved [21,22]. In this paper, we further develop the morphing-based coupling strategy for coupling SPD and CCM models. This is an important step towards the application of coupled formulations, because, to date, no technique has been capable of coupling SPD and CCM models. The fundamental concepts that make morphing-based coupling successful are detailed here.

The remainder of this paper is organized as follows: Section 2 reviews the key formulation of the SPD model; based on the SPD formulation, we develop the linearization of the SPD model based on vector operations in Section 3; Section 4 is devoted to the morphing method between linearized SPD and CCM models; Section 5 presents the finite element discretization of the morphing method; and benchmark examples are shown in Section 6. In addition, conversion formulas between SPD and CCM parameters for homogeneous, isotropic materials are derived and principles of virtual work and minimum potential energy of the SPD model are proved in the Appendices.

2. State-based peridynamic formulation

We list basic definitions in the SPD theory below; readers can find most from [23] with the exception of the different expressions of strain energy density and force state field. We rewrite those expressions as the functions of a full extension of a bond rather than the deviatoric extension [24]. In Section 3, we can then approximate the full extension to get a linearized SPD model.

In this work, we focus on ordinary and elastic materials [23]. Let $\mathcal{H}_\delta(\boldsymbol{x})$ be a spherical neighborhood centered at an arbitrary point $\boldsymbol{x}$ in $\mathbb{R}^3$. Its radius, the horizon, is δ, $\delta > 0$.

Definition 1. Define a reference vector state, $\underline{\mathbf{X}}$, under which the image of a vector $\boldsymbol{\xi} \in \mathcal{H}_\delta(\boldsymbol{x})$ remains itself. That is

$$\underline{\mathbf{X}}\langle\boldsymbol{\xi}\rangle = \boldsymbol{\xi}. \tag{1}$$

Note that a vector $\boldsymbol{\xi} \in \mathcal{H}_\delta(\boldsymbol{x})$ henceforth represents $\boldsymbol{\xi} = \boldsymbol{p} - \boldsymbol{x},\ \forall \boldsymbol{p} \in \mathcal{H}_\delta(\boldsymbol{x})$.