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Bifurcations and catastrophes of a two-degrees-of-freedom nonlinear model simulating the buckling and postbuckling of rectangular plates

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Abstract

The nonlinear buckling and postbuckling behavior of rectangular plates in symmetric and antisymmetric modes is re-examined, in the context of Bifurcation and Catastrophe Theories, using a two-degrees-of-freedom model, which has been adopted for the same purpose in the pioneer literature. At first the perfect system is dealt with in detail, symbolically utilizing the exact as well as the approximate equilibrium equations, the latter being products of a universal unfolding of the original total potential energy function. Conditions for the existence of remote secondary bifurcations are fully assessed and the stability of critical states is determined, revealing sudden qualitative changes in the postbuckling response of the perfect system, which have been also reported for the actual continuous structural system—the rectangular plate—using the von Kárman equations. Thereafter, the imperfection sensitivity is dealt with, introducing symmetric as well as asymmetric imperfections, considered as individual or consecutive perturbations of the perfect system. It is found that symmetry breaking bifurcations give birth to complicated cusp singularities, which may lead to unexpected jumps from one to two-mode remote postbuckling behavior. Finally, considering the general case of random imperfections, higher order two-mode singularities are revealed, mainly of the double-cusp catastrophe type, which have been also discovered in the

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postbuckling response of rectangular plates, a fact validating the choice of the foregoing nonlinear simulation.

Keywords: Buckling and postbuckling; Rectangular plates; Modeling; Bifurcations; Catastrophes; Singularities; Cusps

1. Introduction

One of the main advantages in the application of the methods of bifurcation theory to nonlinear stability problems is that in general it is possible to strongly reduce the dimension of the original (continuous) system to a low dimensional bifurcation system, mainly in the neighborhood of the critical points.

The existence and possibility of such a reduction process follows from experimental evidence and has also been intuitively well-known to engineers for a long time. If, for example, an equilibrium position of an infinite dimensional system turns unstable, or for a small variation of the loading the system exhibits a totally different behavior, often only few (mostly one or two) modes generally determine the qualitative response of the structure dealt with. Evidently, the representation of the nonlinear bifurcation patterns of a continuous system via a simple model of a few degrees of freedom (DOFs) requires that the choice of the model is done in such a way, that the salient features of the real structure are well represented and properly captured. Moreover, even these models may under certain combinations of the foregoing parameters be associated with complicated bifurcation phenomena and a variety of distinct critical and postcritical responses, the study of which—through the simplified simulations—leads to the understanding of the behavior of the corresponding real system, without particular difficulties.

This is the reason why numerous simple mechanical models have been introduced and thoroughly dealt with in the literature, in order to investigate all the indeed amazing multiplicity of static instabilities and bifurcations of structures, summarized in a very comprehensive manner in the pioneer work of Gioncu and Ivan [1]. More specifically, for structures with a distinct postcritical behavior (columns and plates under axial compression), there can be cases associated with remote secondary bifurcations arising in both perfect and imperfect configurations, associated with repetitive changes of the instability forms. In particular the buckling response of orthotropic rectangular plates was initially simulated via a 2-DoFs nonlinear system, developed by Stein (1959) and examined later by Supple and Chilver [2]. The fully nonlinear exact as well as approximate static bifurcation analysis of this particular model, either perfect or imperfect, is the main goal of the present paper, introducing also elements of the Theory of Catastrophes [3–5].

It is found that a variety of secondary bifurcations start from the main postcritical equilibrium configuration and that all related critical branching points in the case of no initial imperfections, under certain combinations of the foregoing parameters, are interconnected, belonging to a remote unstable path in the state-space. The introduction of a single either symmetric or antisymmetric imperfection (perturbation) leads to partial symmetry breaking, associated with independent as well as interconnected singularities of the dual-cusp form, which are all parametrically investigated and presented in detail. Finally, the fully imperfect system is found to exhibit instabilities only of the limit point