Available online at www.sciencedirect.com
ScienceDirect
Journal of the Franklin Institute 344 (2007) 757–764
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

# An adaptive mesh method with variable relaxation time

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Received 8 February 2006; accepted 8 February 2006

**Abstract**

Moving mesh partial differential equations have been widely used in the last decade for solving differential equations exhibiting large solution variations such as shock waves and boundary layers.

In this paper, we have applied a dynamic adaptive method for solving time-dependent differential equations. The mesh velocities are governed by an equation in which a *relaxation time* is employed to move nodes in such a way that they remain concentrated in regions of rapid variation of the solution. A numerical example involving a blow-up problem shows the advantage of using a variable relaxation time over a fixed one.


*Keywords:* Moving mesh method; Blow-up problem; Relaxation time

## 1. Introduction

Adaptive mesh methods have been widely used for approximating partial differential equations that involve large solution variations. Several moving mesh approaches have been derived and many people have discussed the significant improvements in accuracy and efficiency that can be achieved with respect to fixed mesh methods [1–4]. For the type of dynamical moving mesh method considered here, another partial differential equation governing the mesh evolution is solved alongside the original [5,6].

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doi:10.1016/j.jfranklin.2006.02.014

An ideal class of problems for examining the behaviour of moving mesh methods is that of blow-up of temperature in a reacting medium. One of the simplest equations in this class will form the basis of our numerical simulations [7–9]:

$$\begin{aligned} &u_t = u_{xx} + f(u), \\ &u(0,t) = u(1,t) = 0, \\ &u(0,x) = u_0(x). \end{aligned} \tag{1}$$

If $u_0(x)$ is sufficiently large, positive and has a single non-degenerate maximum, then there is a blow-up time $T<\infty$ and a unique blow-up point $x^*$ such that

$$u(x^*,t)\longrightarrow\infty \quad \text{as} \quad t\longrightarrow T$$

and

$$u(x,t)\longrightarrow u(x,T)<\infty \quad \text{if } x\neq x^*.$$

The paper is organized as follows: In Section 2, we review briefly moving mesh methods in which the mesh equations incorporate a relaxation time $\tau$. Blow-up problems are introduced in Section 3 and in Section 4, we improve the moving mesh by describing an extension to the method which uses a variable relaxation time. The numerical experiments in Section 5 illustrate the advantages of this new method.

## 2. Moving mesh methods

Let $x$ and $\xi$ denote the physical and computational coordinates, respectively, both of which are assumed to be in [0, 1]. Define a one-to-one coordinate transformation by

$$\begin{aligned} &x = x(\xi,t), \quad \xi\in[0,1], \\ &x(0,t) = 0, \quad x(1,t) = 1. \end{aligned}$$

The computational coordinate is discretized on a uniform mesh given by

$$\xi_i = \frac{i}{N}, \quad i = 0,1,\ldots,N, \tag{2}$$

where $N$ is a certain positive integer and the corresponding non-uniform mesh in $x$ is denoted by

$$0 = x_0 < x_1(t) < x_2(t) < \cdots < x_{N-1}(t) < x_N = 1.$$

For a chosen monitor function $M(x,t)>0$, the moving mesh $x_i(t)$ satisfies the following equidistribution principle (EP) for all values of time $t$. The equidistribution principle is one of the most important concepts in the development of moving mesh methods [1]:

$$\int_{x_{i-1}(t)}^{x_i(t)} M(x,t)\,\mathrm{d}x = \frac{1}{N}\int_0^1 M(x,t)\,\mathrm{d}x = \frac{\theta(t)}{N}$$

or

$$\int_0^{x_i(t)} M(x,t)\,\mathrm{d}x := \frac{i}{N}\theta(t) = \xi_i\theta(t). \tag{3}$$