Available online at www.sciencedirect.com
ScienceDirect

Journal of the Franklin Institute 344 (2007) 790–799

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

Codes over Galois rings with respect to the Rosenbloom–Tsfasman metric

Mehmet Ozen[a], Irfan Siap[b,*]

[a]Department of Mathematics Sakarya, Sakarya University, Turkey
[b]Adıyaman Education Faculty, Gaziantep University, Adıyaman, Turkey

Received 1 February 2006; accepted 1 February 2006

Abstract

We investigate the structure of codes over Galois rings with respect to the Rosenbloom–Tsfasman (shortly RT) metric. We define a standard form generator matrix and show how we can determine the minimum distance of a code by taking advantage of its standard form. We compute the RT-weights of a linear code given with a generator matrix in standard form. We define maximum distance rank (shortly MDR) codes with respect to this metric and give the weights of the codewords of an MDR code. Finally, we give a decoding technique for codes over Galois rings with respect to the RT metric.

Keywords: Linear codes; Galois rings; Rosenbloom–Tsfasman metric

1. Introduction

The RT (non-Hamming) metric for codes over fields is defined in [1] and some bounds for the minimum distance are established. Some applications of this metric to uniform distributions are given in [2]. A MacWilliams identity for codes over matrices with respect to the RT metric is proven in [3]. Also later, another MacWilliams identity for complete weight enumerators of codes over matrices with respect to the RT metric is proven in [4]. The structure and decoding of linear codes over fields with respect to the RT metric have been investigated in [5]. In [6] the structure and decoding of linear codes over the ring $\mathbb{F}_q[u](u^s)$ with respect to the RT metric is explored. In this paper, we investigate the structure of codes over Galois rings. We give a formula for RT-weights of codewords of a

*Corresponding author.
E-mail addresses: ozen@sakarya.edu.tr (M. Ozen), isiap@gantep.edu.tr (I. Siap).

0016-0032/$30.00
doi:10.1016/j.jfranklin.2006.02.001

linear code given with a generator matrix in standard form. Later, we define maximum distance rank (MDR) codes and compute the spectra of the weights of MDR linear codes. Finally, we give a decoding technique for linear codes over integers modulo p^e and finally for Galois rings with respect to the RT metric.

Let $R = GR(p^e, t)$ denote a finite Galois ring of characteristic p^e and cardinality p^{et}, where p is a prime number and e, t are positive integers. Detailed and further information regarding Galois rings can be found in [7,8].

The Hamming weight of a codeword $\boldsymbol{u}$ which is defined by $w(\boldsymbol{u}) = |\{i \mid u_i \neq 0\}|$, i.e. the number of the nonzero entries of $\boldsymbol{u}$. The minimum weight $w(C)$ of a code C is the smallest possible weight among all its nonzero codewords. We observe that if C is a linear code then $d(C) = w(C)$.

A linear code C over a finite field is called a t-error correcting code if $t = \lfloor (d-1)/2 \rfloor$, where $d = d(C)$. Further information regarding Hamming codes over fields can be found in [9].

Let $\boldsymbol{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$. The RT weight of $\boldsymbol{x}$ is defined by

$$w_{\mathrm{N}}(\boldsymbol{x}) = \begin{cases} \max\{i \mid x_i \neq 0\}, & \boldsymbol{x} \neq 0, \\ 0, & \boldsymbol{x} = 0. \end{cases}$$

The RT metric (ρ distance) is defined by $\rho(\boldsymbol{x}, \boldsymbol{y}) = w_{\mathrm{N}}(\boldsymbol{x} - \boldsymbol{y})$, where $\boldsymbol{x}, \boldsymbol{y} \in R^n$.

Note the difference between the weight and distance notations of Hamming and RT (non-Hamming) metrics. The letter "N" for RT metric is used to emphasize the non-Hamming case.

Definition 1.1. An R submodule C of R^n is called an R-linear code.

Definition 1.2. Let C be an R-linear code. The minimum nonzero ρ distance between the codewords of C is denoted by $d_{\mathrm{N}}(C)$. The minimum nonzero ρ weight among all codewords of C is denoted by $w_{\mathrm{N}}(C)$. In linear case, $d_{\mathrm{N}}(C) = w_{\mathrm{N}}(C)$, and $d_{\mathrm{N}}(C)$ is called the minimum distance of C with respect to the RT metric.

Theorem 1.1 (*Huffman [10]*). *In Hamming case, a generator matrix of an $R = GR(p^e, t)$-linear code C, is equivalent to a linear code generated by*

$$A = \begin{pmatrix} I_{k_1} & A_{11} & A_{12} & \cdots & A_{1e} \\ 0 & pI_{k_2} & A_{22} & \cdots & pA_{2e} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & p^{e-1}I_{k_e} & p^{e-1}A_{ee} \end{pmatrix}, \tag{1}$$

where A_{ij}'s denote matrices whose entries are from R and $I_{k_1}, I_{k_2}, \ldots, I_{k_e}$ are identity matrices of sizes $k_1, k_2, \ldots, k_e$, respectively. The number of elements of C is equal to $p^{t\alpha}$, where $\alpha = \sum_i^e k_i(e-i+1)$.

A linear code C generated by a matrix (1) is called a $(k_1, k_2, \ldots, k_e)$-type code.