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A new framework for large strain electromechanics based on convex multi-variable strain energies: Variational formulation and material characterisation

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Abstract

Following the recent work of Bonet et al. (2015), this paper postulates a new convex multi-variable variational framework for the analysis of Electro Active Polymers (EAPs) in the context of reversible nonlinear electro-elasticity. This extends the concept of polyconvexity (Ball, 1976) to strain energies which depend on non-strain based variables introducing other physical measures such as the electric displacement. Six key novelties are incorporated in this work. First, a new definition of the electro-mechanical internal energy is introduced expressed as a convex multi-variable function of a new extended set of electromechanical arguments. Crucially, this new definition of the internal energy enables the most accepted constitutive inequality, namely ellipticity, to be extended to the entire range of deformations and electric fields and, in addition, to incorporate the electro-mechanical energy of the vacuum, and hence that for ideal dielectric elastomers, as a degenerate case. Second, a new extended set of variables, work conjugate to those characterising the new definition of multi-variable convexity, is introduced in this paper. Third, both new sets of variables enable the definition of novel extended Hu–Washizu type of mixed variational principles which are presented in this paper for the first time in the context of nonlinear electro-elasticity. Fourth, some simple strategies to create appropriate convex multi-variable energy functionals (in terms of convex multi-variable invariants) by incorporating minor modifications to a priori non-convex multi-variable functionals are also presented. Fifth, a tensor cross product operation (de Boer, 1982) used in Bonet et al. (2015) to facilitate the algebra associated with the adjoint of the deformation gradient tensor is incorporated in the proposed variational electro-mechanical framework, leading to insightful representations of otherwise complex algebraic expressions. Finally, under a characteristic experimental setup in dielectric elastomers, the behaviour of a convex multi-variable constitutive model capturing some intrinsic nonlinear effects such as electrostriction, is numerically studied.

Keywords: Dielectric elastomers; Nonlinear electro-elasticity; Large strains; Material stability; Mixed variational principles; Polyconvexity

1. Introduction

The actuator and harvesting capabilities of the early piezoelectric crystals and ceramics were not very long ago eclipsed by those of Electro Active Polymers (EAPs). This heterogeneous group can be divided into two further subgroups, namely Electronic Electro Active Polymers (EEAPs) and Ionic Electro Active Polymers (IEAPs) [1].

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Within the first subgroup, Dielectric Elastomers (DEs) and electrostrictive relaxor ferroelectric polymers or simply Piezoelectric Polymers (PPs), have become increasingly relevant. The second subgroup includes ionic gels, Ionic Polymer Metal Composites (IPMCs) and carbon nanotubes.

The present manuscript focuses mainly on DEs and PPs. The elastomer VHB 4910 and the highly popular PolyVinylidene DiFlouride (PVDF) are the most representative examples of both groups, respectively. However, DEs are becoming specially attractive due to their outstanding actuation properties [2–5]. For instance, a voltage induced area expansion of 1980% on a DE membrane film has been recently reported by Li et al. [6]. In this specific case, the electromechanical instability is harnessed as a means for obtaining these electrically induced massive deformations with potential applications in soft robots, adaptive optics, balloon catheters and Braille displays [6], among others. Moreover, these materials have been successfully applied as generators to harvest energy from renewable sources as human movements and ocean waves [7].

With the emergence of these highly deformable materials, a variational framework for nonlinear electro-elasticity was developed by different authors [8–18]. Within that framework, as customary in nonlinear continuum mechanics, the constitutive behaviour of the material is encoded in an energy functional which depends typically upon appropriate strain measures, a Lagrangian electric variable and, if dissipative effects are considered, upon an electromechanical internal variable.

Several authors have proposed alternative representations of the energy functional in terms of electromechanical invariants [9–14]. However, some restrictions need to be imposed on the invariant representation if physically admissible behaviours are expected to occur. Bustamante and Merodio [19] considered classical constitutive inequalities, namely: the Baker–Ericksen inequality, the pressure–compression inequality, the traction–extension inequality and the ordered forces inequality. The objective was to study under what conditions a specific invariant representation of the energy functional for magneto-sensitive elastomers would violate the previous inequalities for very specific deformation scenarios.

The most well accepted constitutive inequality is ellipticity, also known as the Legendre–Hadamard condition [20,21]. This inequality has important physical implications. In particular, it guarantees positive definiteness of the generalised electromechanical acoustic tensor [22] and hence, existence of real wave speeds in the material in the vicinity of an equilibrium configuration. Several authors have also studied under what conditions positive definiteness of the generalised electromechanical acoustic tensor is compromised for a specific invariant representation of the energy functional [22–24]. Recently, a material stability criterion based on an incremental quasi-convexity condition of the energy functional has been introduced by Miehe et al. [25].

In nonlinear elasticity, polyconvexity [20,21,26–44] of the (strain) energy functional, namely, convexity with respect to the components of the deformation gradient tensor $\boldsymbol{F}$, the components of its adjoint or cofactor $\boldsymbol{H}$ and its determinant J, automatically implies the ellipticity condition. Following the work of Rogers [45], the present manuscript presents an extension of the concept of polyconvexity to the field of nonlinear electro-elasticity based on a new convex multi-variable definition of the energy functional. Notice that the focus of this paper is on material stability and not on the existence of minimisers. The latter would also require the study of the sequentially weak lower semicontinuity and the coercivity of the energy functional.

A new electro-kinematic variable set is introduced including the deformation gradient $\boldsymbol{F}$, its adjoint $\boldsymbol{H}$, its determinant J, the Lagrangian electric displacement field $\boldsymbol{D}_0$ and an additional spatial or Eulerian vector $\boldsymbol{d}$ computed as the product between the deformation gradient tensor and the Lagrangian electric displacement field. The resulting energy functional is called the internal energy and as presented in Ref. [46], convexity of the internal energy functional with respect to the elements of the new extended set permits an extension of the concept of ellipticity [21], not only to the entire range of deformations but to any applied electric field as well.

The extended set of variables $\mathcal{V} = \{\boldsymbol{F}, \boldsymbol{H}, J, \boldsymbol{D}_0, \boldsymbol{d}\}$ enables the introduction of another new set of work conjugate variables $\Sigma_{\mathcal{V}} = \{\boldsymbol{\Sigma}_{\boldsymbol{F}}, \boldsymbol{\Sigma}_{\boldsymbol{H}}, \Sigma_J, \boldsymbol{\Sigma}_{\boldsymbol{D}_0}, \boldsymbol{\Sigma}_{\boldsymbol{d}}\}$ [47]. Convexity of the internal energy with respect to $\boldsymbol{\mathcal{V}}$ guarantees that the relationship between both sets of variables is one to one and invertible. In addition, convexity of the internal energy enables three additional energy functionals to be defined (at least implicitly) by making appropriate use of the Legendre transform.[1]

[1] Two partial Legendre transforms can be obtained by fixing either purely mechanical or purely electrical variables of the extended set. A total Legendre transform of the internal energy would render the third energy functional in terms of the elements of the extended set of work conjugate variables.