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Hopf bifurcation control for delayed complex networks ☆

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Abstract

In this paper, we consider the problem of Hopf bifurcation control for a complex network model with time delays. We know that for the system without control, as the positive gain parameter of the system passes a critical point, Hopf bifurcation occurs. To control the Hopf bifurcation, a time-delayed feedback controller is proposed to delay the onset of an inherent bifurcation when such bifurcation is undesired. Furthermore, we can also change the stability and direction of bifurcating periodic solutions by choosing appropriate control parameters. Numerical simulation results confirm that the new feedback controller using time delay is efficient in controlling Hopf bifurcation.

Keywords: Hopf bifurcation; Periodic solution; Complex networks; Bifurcation control; Numerical simulation

1. Introduction

A great deal of research interest of complex networks have been aroused since the work of Watts and Strogatz [1]. From then on, much attention has been paid to the dynamics of complex networks and complex networks has been discovered in the Internet, the World Wide Web (WWW), the World Trade Web, linguistic webs, food webs etc. [2–7]. Complex

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networks have also found numerous applications in various fields such as physics, technology, and the life sciences [8–11].

Recently, complex networks with diseases spreading are proposed and their dynamical properties have been analyzed. In particular, Moukarzel [3], and Newman and Watts [6] studied a linear disease-spreading model in small-world networks. Yang [7] introduced nonlinear friction effects on the linear spreading models with time-delays, which results in the discoveries of chaos in small-world networks. To describe the effect of the new link-adding probability p on the stability and bifurcation behaviors of disease spreading in N–W networks, Li [9] proposed the model as

$$\frac{\mathrm{d}V(t)}{\mathrm{d}t} = 1 + 2pV(t-\delta) - \mu(1+2p)V^2(t-\delta), \tag{1}$$

where V is the total influenced volume, μ is a measure of nonlinear interactions in the network and p is the probability of add linkages between pairs of randomly chosen nodes. The model contain regular lattices with $p = 0$, small-world networks with $0 < p \ll 1$ and random networks with $p = 1$. Local stability and existence of Hopf bifurcation in the new model were studied, along with some conditions on the directions and stabilities of the bifurcating periodic solutions.

In this paper, bifurcation control using a time-delayed feedback controller for the complex network model (1) will be considered. Bifurcation control refers to the task of designing a controller to suppress or reduce some existing bifurcation dynamics of a given nonlinear system, thereby achieving some desirable dynamical behaviors [12]. Typical bifurcation control objectives include delaying the onset of an inherent bifurcation, changing the parameter value of an existing bifurcation point. In recent years, bifurcation control [12–18], periodic solutions [19–21] and synchronization [23–25] on delayed system have attracted many researchers from various disciplines. We will show, with a Hopf bifurcation controller, that one can increase the critical value of positive parameter. Furthermore, we can also change the stability and direction of bifurcating periodic solutions by choosing appropriate parameters.

The remainder of this paper is organized as follows. The existence of Hopf bifurcation parameter is determined in Section 2. In Section 3, based on the normal form method and the center manifold theorem introduced by Hassard et al. [22], the direction, orbitally stability and the period of the bifurcating periodic solutions are analyzed. To verify the theoretic analysis, numerical simulations are given in Section 4. Finally, Section 5 concludes with some discussions.

2. Existence of Hopf bifurcation

In this section, we focus on designing a time-delayed feedback controller in order to control the Hopf bifurcation arising from the complex network model. The following conclusions for the uncontrolled system (1) are needed at first [9]:

Lemma 1. *If* $\delta < \pi/(4p)$, *then when the positive parameter* μ *passes through the critical value* $\mu^* = (\pi^2 - 16\delta^2 p^2)/(16\delta^2(1+2p))$, *there is a Hopf bifurcation in model* (1) *at its equilibrium* $V^* = (p + \sqrt{p^2 + \mu(1+2p)})/(\mu(1+2p))$.

We now turn to study how to control the Hopf bifurcation to achieve desirable behaviors through control parameters. The time-delayed feedback controller are designed