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A new framework for large strain electromechanics based on convex multi-variable strain energies: Finite Element discretisation and computational implementation

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Abstract

In Gil and Ortigosa (2016), Gil and Ortigosa introduced a new convex multi-variable framework for the numerical simulation of Electro Active Polymers (EAPs) in the presence of extreme deformations and electric fields. This extends the concept of polyconvexity to strain energies which depend on non-strain based variables. The consideration of the new concept of multi-variable convexity guarantees the well posedness of generalised Gibbs' energy density functionals and, hence, opens up the possibility of a new family of mixed variational principles. The aim of this paper is to present, as an example, the Finite Element implementation of two of these mixed variational principles. These types of enhanced methodologies are known to be necessary in scenarios in which the simpler displacement-potential based formulation yields non-physical results, such as volumetric locking, bending and shear locking, pressure oscillations and electro-mechanical locking, to name but a few. Crucially, the use of interpolation spaces in which some of the unknown fields are described as piecewise discontinuous across elements can be used in order to efficiently condense these fields out. This results in mixed formulations with a computational cost comparable to that of the displacement-potential based approach, yet far more accurate. Finally, a series of very challenging numerical examples are presented in order to demonstrate the accuracy, robustness and efficiency of the proposed methodology.

Keywords: Electro active polymers; Nonlinear electro-elasticity; Polyconvexity; Mixed variational principles; Material stability; Finite Elements

1. Introduction

The present manuscript presents a Finite Element computational framework tailor-made for the simulation of Electro Active Polymers (EAPs) [1–6] in applications where very large deformations and electric fields are involved. Dielectric elastomers and piezoelectric polymers are examples of EAPs which can be subjected to these extreme scenarios. For instance, giant electrically induced deformations of 1980% have been reported in the experimental literature [7] for the most representative example of dielectric elastomers, namely, the elastomer VHB 4910. Moderate electrically induced deformations of 40% have been reported in piezoelectric polymers, such as the highly popular PolyVinylidene DiFlouride (PVDF).

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Several authors [8–13] have contributed to the development of variational approaches in the context of nonlinear electro-elasticity. Very importantly in this case, the constitutive laws governing the physics of the coupled problem must satisfy physically meaningful constitutive inequalities [14]. Bustamante and Merodio [15] studied under what ranges of deformation and magnetic field the Baker–Ericksen inequality [14] would be compromised, specifically considering smart materials belonging to the class of magneto-sensitive elastomers.[1] Recently, a material stability criterion based on an incremental quasi-convexity condition of the energy functional has been introduced by Miehe et al. [16].

In Ref. [17], Gil and Ortigosa, following the work of Rogers [18], extend the concept of polyconvexity [14,19–26] to the field of nonlinear electro-elasticity based on a new convex multi-variable definition of the energy functional. It should be emphasised that the new definition of multi-variable convexity ensures [17] the material stability and well posedness of the equations. The existence of minimisers would also require the study of the sequentially weak lower semicontinuity and the coercivity of the energy functional. The internal energy density is defined as a convex multi-variable function of a new electro-kinematic variable set, including the deformation gradient $\boldsymbol{F}$, its adjoint $\boldsymbol{H}$, its determinant J, the Lagrangian electric displacement field $\boldsymbol{D}_0$ and an additional spatial or Eulerian vector $\boldsymbol{d}$ computed as the product between the deformation gradient tensor and the Lagrangian electric displacement field. This definition enables the most accepted constitutive inequality, namely ellipticity (rank-one convexity) [14], to be fulfilled for the entire range of deformations and electric fields. Moreover, taking advantage of a new tensor cross product operation [24,27], tedious algebra is remarkably simplified yielding insightful representations of otherwise complex expressions (i.e. electro-mechanical tangent operators).

Crucially, the new multi-variable convexity criterion enables the incorporation of the internal energy of the vacuum as a degenerate case [17]. In addition, the consideration of convex multi-variable internal energy functionals leads to positive definiteness of the generalised electro-mechanical acoustic tensor [28–30] and hence, existence of real wave speeds in the material in the vicinity of an equilibrium configuration. This analysis is shown in Ref. [31] as a by-product of the hyperbolicity of a generalised system of conservation laws, along the same lines as Refs. [21,32–38].

From the computational standpoint, most authors [39–43] tend to prefer Finite Element computational frameworks where only geometry and electric potential are part of the solution. However, it is well known that these types of formulations can produce non-physical results, such as volumetric locking, bending and shear locking, pressure oscillations and electro-mechanical locking, to name but a few [44–46].

The consideration of convex multi-variable energy functionals brings additional benefits, including the solution to above shortcomings. Indeed, the extended set of variables $\{\boldsymbol{F}, \boldsymbol{H}, J, \boldsymbol{D}_0, \boldsymbol{d}\}$ enables the introduction of work conjugates $\{\boldsymbol{\Sigma}_{\boldsymbol{F}}, \boldsymbol{\Sigma}_{\boldsymbol{H}}, \Sigma_J, \boldsymbol{\Sigma}_{\boldsymbol{D}_0}, \boldsymbol{\Sigma}_{\boldsymbol{d}}\}$, where the satisfaction of multi-variable convexity guarantees that the relationship between both sets is one to one and invertible. Based on this, a new family of extended Hu–Washizu type of variational principles [22,47–54] can be introduced [17]. The development of these new mixed variational principles [17] is a robust alternative to resolve the spurious (non-physical) mechanisms associated to the more classical displacement-potential approach.

This paper is organised as follows. Section 2 briefly introduces the governing equations of the problem, with special emphasis in the extension of the concept of polyconvexity from the field of convex multi-variable nonlinear elasticity to the more general field of nonlinear electro-elasticity [17], where the basic ingredients of the new framework are presented. In addition, a series of generalised Gibbs' energy density functionals is presented, defined through the use of Legendre transformations. Section 3 presents two of the extended Hu–Washizu type of mixed variational principles presented in Ref. [17], which will be the main objective of this paper. Section 4 is focused on the Finite Element discretisation and implementation of the variational principles presented in previous Section 3. Section 5 includes a wide spectrum of challenging numerical examples in order to demonstrate the robustness and applicability of the proposed enhanced mixed formulations, ranging from simpler isotropic to more complex anisotropic convex multi-variable models. Finally, Section 6 provides some concluding remarks and a summary of the key contributions of this paper.

Three appendices have been included for the sake of completeness. Appendix A shows how to relate the various expressions of the tangent operator of the internal energy, when expressed both in terms of the displacements and electric displacement field and when expressed in terms of the extended set of electro-kinetic variables $\{\boldsymbol{F}, \boldsymbol{H}, J, \boldsymbol{D}_0, \boldsymbol{d}\}$. Appendix B establishes the relationship between the classical components of the Helmholtz's energy and those associated

[1] There exists a clear similitude between magneto-elasticity and electro-elasticity.