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Triangular Mindlin microplate element

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Abstract

In this article, based on the most general form of strain gradient theory (MGSGT), a novel extended triangular Mindlin plate element is proposed. To accomplish this aim, first, the quadratic form of energy functional is obtained by vectorizing the higher-order tensors of energy pairs, from which the stiffness and mass matrices of the element are readily derived. In comparison with the standard Mindlin plate element, the new element needs three additional nodal degrees of freedom (DOF) including derivatives of lateral deflection and rotations, which means a total of nine DOFs per node. Also, as compared to the standard Mindlin plate element which requires only C^0 shape functions, the present one requires C^1 continuous smooth shape functions due to second derivatives of deflection and rotations. Hence, cubic polynomials are used to interpolate the displacement components. The new element can be reduced to that based on the modified strain gradient theory (MSGT) and the modified couple stress theory (MCST). Moreover, the standard Mindlin plate element is recovered when the gradient-based material parameters tend to zero. The Mindlin microplates with different boundary conditions are considered as the problem under study whose free vibration and bending are analyzed. The results are compared with the exact solutions and excellent agreement is achieved.

Keywords: Finite element method; Triangular element; Strain gradient theory; Mindlin plate theory

1. Introduction

Microbeams and microplates are widely used in different fields such as in biotechnology [1], radiology [2], resonators [3], microbiology [4], radioactivity detection [5], and in many microelectromechanical systems (MEMS) such as micro-pressure sensors [6,7], micro-pumps [8,9] and micro-switches [10]. Therefore, different characteristics of these microstructures, especially their mechanical behavior, have been the focus of significant research. In this regard, classic continuum models have been used in different research works to analyze the mechanical problems of microbeams and microplates (e.g., free vibration and buckling in both linear and nonlinear regimes) [11–21].

Since it is generally accepted that the mechanical behavior of microstructures is dependent on size [22–26], one can conclude that the application of conventional continuum elasticity, which is scale-free, may be inappropriate at micro-scale. Hence, some higher-order elasticity theories have been developed that are capable of accommodating size effects. Of these size-dependent continuum theories, one can mention strain gradient theory (SGT) [27,28], modified strain gradient theory (MSGT) [29], couple stress theory (CST) [30,31] and modified couple stress theory (MCST) [32]. Many investigators successfully employed such theories in order to study the bending [33–37],

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buckling [38–41], vibration [42–47] and pull-in [48–52] phenomena in microstructures. For instance, Ke et al. [34] developed a MCST Mindlin plate model to study the bending, buckling and free vibration of annular microplates made of functionally graded materials (FGMs). Based on the most general form of strain gradient theory (MGSGT), Ansari and his co-workers [36] investigated the bending, buckling and free vibration of FGM Timoshenko microbeams. Akgöz and Civalek [39] used SGT and MCST so as to analyze the axial buckling of Euler–Bernoulli microbeams. A MCST Euler–Bernoulli beam model was used by Kahrobaiyan and his associates [45] in order to study the nonlinear forced vibration of microbeams. Ramezani [46] utilized SGT to investigate the nonlinear free vibration of Kirchhoff microplates. Zhang and Fu [50] presented the pull-in analysis of electrically actuated viscoelastic microbeams on the basis of MCST. Ansari et al. [52] studied the pull-in phenomenon in hydrostatically and electrostatically actuated circular microplates using MSGT.

The governing equations and boundary conditions of a size-dependent model are more complicated than those of its classic counterpart. Thus, seeking analytical solutions for the problems of microstructures is not possible in many cases and numerical techniques must be used for the solution procedure. Among different numerical techniques, the finite element method (FEM) is one of the most powerful tools for the analysis of structures. Some researchers used FEM to analyze the mechanical behavior of microbeams and microplates. For example, Roux et al. [53] used FEM to study an electro-optical micro-shutter. Lai and Chang [54] designed and analyzed radio-frequency (RF) MEMS micro-switches with different geometries based upon FEM. Ahmadian et al. [55] applied corotational FEM for dynamic modeling of geometrically nonlinear electrostatically actuated microbeams. Recently, Ricci et al. [56] employed FEM for the frequency spectrum estimation of a resonating microplate in a microfluidic chamber. All of the aforementioned FEM analyses were performed according to the classic continuum mechanics.

Based on the presented subjects so far, it is concluded that developing size-dependent finite element approaches on the basis of higher-order elasticity theories can be promising in the field of micro- and nano-mechanics because of two reasons. First, it is necessary to adopt higher-order elasticity theories when describing the mechanical behavior of structures at very small scales. Second, it is sometimes difficult, if not impossible, to obtain analytical exact solutions to the problems arising from modeling microstructures based on the higher-order theories. Motivated by this consideration, some researchers have developed non-classic finite element models based on the size-dependent elasticity theories. For example, Ansari et al. [57,58] proposed size-dependent finite element approaches for the linear and nonlinear analyses of Timoshenko microbeams based on the strain gradient theory. They indicated that their developed non-classic microbeam elements are able to effectively predict the size-dependent mechanical behaviors of microbeams.

In the present work, a new size-dependent triangular plate element is developed which is capable of accommodating the size effect in order to describe the mechanical behavior of microplates. The five material length scale parameters are considered on the basis of Mindlin's strain gradient theory [27]. The Mindlin plate theory is also utilized for the plate modeling. The rest of this article has been arranged as follows:

In Section 2, the strain gradient theory is described and its fundamental relations are presented. In Section 3, the classic and higher-order stresses and strains in the plate model are matricized. In Section 4, the finite element relations of the triangular element including strain gradient effects are explained. To obtain the relations of the element, the higher-order tensors of energy pairs in the energy functional are vectorized and are then written in the quadratic form. Since derivatives of lateral deflection and rotations are considered as nodal DOFs in the present plate element, it has three additional DOFs per node as compared to the standard plate element. Moreover, due to the presence of second derivatives of deflection and rotations, the new element needs C^1 continuous smooth shape functions. Therefore, cubic polynomials are used for the interpolation purpose. It is shown that for some specific values of length scale parameters, the formulation of the element can be simply reduced to that based on MSGT, MCST and classic theory. In Section 5, the free vibration and bending problems of microplates under various types of boundary conditions are addressed using the developed element. The convergence and validity of the results are indicated. For the comparison purpose, an exact solution is presented for the vibrations of simply-supported microplates based on MSGT. Furthermore, selected results are given to examine the strain gradient effects on the vibrations and bending of microplates.

2. Strain gradient theory

In the elasticity theory, the strain energy is introduced as

$$U = \int_{\Omega} \hat{U} \, dV \tag{1}$$