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Approximations to the solution of linear Fredholm integrodifferential–difference equation of high order

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Abstract

There are few techniques available to numerically solve linear Fredholm integrodifferential-difference equation of high-order. In this paper we show that the Taylor matrix method is a very effective tool in numerically solving such problems. This method transforms the equation and the given conditions into the matrix equations. By merging these results, a new matrix equation which corresponds to a system of linear algebraic equation is obtained. The solution of this system yields the Taylor coefficients of the solution function. Some numerical results are also given to illustrate the efficiency of the method. Moreover, this method is valid for the differential, difference, differential–difference and Fredholm integral equations. In some numerical examples, MAPLE modules are designed for the purpose of testing and using the method.

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1. Introduction

The complexity of the questions encountered in the study of differential, difference and integral equations is not mitigated in the analysis of integrodifferential–difference equations, a combination of these subjects. The mentioned equations arise in great many branches of sciences. Also, these equations occure frequently as a model in mathematical

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biology and the physical sciences. For example, the integrodifference equations given in [1, pp. 304, 320] describe the efflux of gas from the open end of a tube and the integrodifferential–difference equation given by the integral equation of Palm. [1, p. 304] arises in queueing theory. Here, to obtain the solution of these equations, Laplace transform methods are applied. Another example, the integrodifferential equation given in [2] is a good model in one-dimensional viscoelasticity and also a model for circulating fuel nuclear reactor.

In recent years, the studies of differential–difference equations, i.e. equations containing shifts of the unknown function and its derivatives, are developed very rapidly and intensively [2–7]. Problems involving these equations arise in studies of control theory [6], in determining the expected time for the generation of action potentials in nerve cells by random synaptic inputs in the dendrites [5], in the modelling of the activation of a neuron [5], in the works on epidemics and population [1], in the two-body problems in the physical systems whose acceleration depends upon its velocity and its position at earlier instants, and in the formulation of the biological reaction phenomena to X-rays [1].

On the other hand, a Taylor method for solving Fredholm integral equation has been presented by Kenwall and Liu [8] and then this method has been extended by Sezer to Fredholm integro-differential equations [9] and second-order linear differential equations [10].

In this study, the basic ideas of the above studies are developed and applied to the mth-order linear Fredholm integrodifferential–difference equation with variable coefficients

$$\sum_{k=0}^{m} P_k(x)y^{(k)}(x) + \sum_{r=0}^{n} P_r^*(x)y^{(r)}(x-\tau) = f(x) + \int_a^b K(x,t)y(t-\tau)\,\mathrm{d}t \quad \tau \geqslant 0, \tag{1}$$

which is extended of differential–differences equations given in [1, Section 7] with the mixed conditions

$$\sum_{k=0}^{m-1} \left[a_{ik}y^{(k)}(a) + b_{ik}y^{(k)}(b) + c_{ik}y^{(k)}(c)\right] = \mu_i \tag{2}$$

$i = 0(1)(m-1)$, $a \leqslant c \leqslant b$ and the solution is expressed in the form

$$y(x) = \sum_{n=0}^{N} \frac{y^{(n)}(c)}{n!}(x-c)^n, \quad a \leqslant c \leqslant b, \quad N \geqslant m, \tag{3}$$

which is a Taylor polynomial of degree N at $x = c$, where $y^{(n)}(c)$, $n = 0(1)N$ are the coefficients to be determined.

Here $P_k(x)$, $P_r^*(x)$, $K(x,t)$ and $f(x)$ are functions defined on $a \leqslant x \leqslant b$ and can be expanded to the Taylor series about $x = c$; the real coefficients a_{ik}, c_{ik}, b_{ik} and μ_i are appropriate constants.

The rest of this paper is organized as follows. The fundamental relations related with high-order linear Fredholm integrodifferential–difference equation with variable coefficients are presented in Section 2. The new scheme is based on Taylor matrix method.