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Proper generalized decomposition for parameterized Helmholtz problems in heterogeneous and unbounded domains: Application to harbor agitation

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Highlights

- Novel parameterized solutions for a scattering problem with the proper generalized decomposition (PGD).
- Development and viability of PGD for non-Hermitian operator in an unbounded and heterogeneous domain.
- Adapt the perfectly matched layer approach for artificial boundaries to be used within a PGD scheme.
- Formalization of the higher-order PGD-projection to obtain an optimal separable representation.
- Comparison of the higher-order PGD-projection with High Order Singular Value Decomposition (HOSVD).

Abstract

Solving the Helmholtz equation for a large number of input data in an heterogeneous media and unbounded domain still represents a challenge. This is due to the particular nature of the Helmholtz operator and the sensibility of the solution to small variations of the data. Here a reduced order model is used to determine the scattered solution everywhere in the domain for any incoming wave direction and frequency. Moreover, this is applied to a real engineering problem: water agitation inside real harbors for low to mid-high frequencies.

The proper generalized decomposition (PGD) model reduction approach is used to obtain a separable representation of the solution at any point and for any incoming wave direction and frequency. Here, its applicability to such a problem is discussed and demonstrated. More precisely, the contributions of the paper include the PGD implementation into a perfectly matched layer framework to model the unbounded domain, and the separability of the operator which is addressed here using an efficient higher-order projection scheme.

Then, the performance of the PGD in this framework is discussed and improved using the higher-order projection and a Petrov–Galerkin approach to construct the separated basis. Moreover, the efficiency of the higher-order projection scheme is demonstrated and compared with the higher-order singular value decomposition.

Keywords: Reduced order models; Proper generalized decomposition; Helmholtz; Parameterized solutions; Wave propagation; Harbor

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1. Introduction

A large number of models involving the propagation of harmonic waves in unbounded domains are governed by Helmholtz-type partial differential equations. Their numerical solution is usually computationally demanding. Well-known difficulties are: pollution errors [1–3], treatment of the unbounded domain [4], and modeling small geometric features that have a large influence on the scattered field [5,6]. Moreover, in engineering practice, wave propagation computations are usually one of many steps in a design process, an optimization strategy or an identification analysis. In summary, accuracy is compromised because the large computer costs drastically reduce the number (or range) of parameters tested. Note that the obvious approach of directly interpolating a few (costly) computed solutions to estimate results for intermediate parameter values is not viable because the solution is extremely sensitive to the parameters (e.g. incoming wave frequency and direction, geometry, etc.).

This paper proposes a strategy to reduce the computational limit imposed on the number of Helmholtz solutions that are feasible to compute in a design or optimization process. More precisely, the objective is to construct the generalized (high-dimensional) solution of a parameterized scattering problem in an heterogeneous and unbounded domain. This generalized solution, recently called *computational vademecum* [7] in a more general framework, provides the engineer a way to evaluate in real-time any tentative scattering situation (e.g. the Helmholtz solution and its derivatives). Therefore it extremely accelerates the process of evaluating solutions of the Helmholtz problem.

There are several possibilities to parameterize the scattering problem, here the focus is on the parameters defining the incident wave: angular frequency and incoming wave direction. Each of these parameters ranges in a bounded interval (usually application-dependent) and is considered as a new 1D coordinate of the classical Helmholtz equation. This results in a high-dimensional Helmholtz problem whose solution provides the scattered field at any point of the domain and for any incident condition. Moreover, the generalized solution is computed only once whenever it is assumed that the other data (geometry, boundary conditions, etc.) do not change, which is usual in most of engineering applications. The important point is that any subsequent evaluation of the scattered field is readily obtained by means of a fast post-process (this, for instance, can be the case of a time signal including a wide range of frequencies).

The high-dimensional character of the proposed problem involves an exponential growth of degrees of freedom (the so-called *curse of dimensionality*) when using standard mesh-based discretization techniques. A reduced order model can circumvent this critical difficulty, see some previous applications to parameterized Helmholtz equations in [8–12], among others.

Here, the proper generalized decomposition (PGD) [13,14] is used. This method has been studied and successfully applied to various problems in computational mechanics, see [15–19]. The interested reader is addressed to [20–22] and the references therein for a survey on different PGD techniques. PGD computes iteratively each term of the approximation using products of functions defined in lower dimensions and induces a separated representation (approximation) of the solution. This reduces the high-dimensional complexity of the original problem. Therefore, it is able to circumvent the curse of dimensionality and provide an approximation of the generalized solution.

As usual in any reduced order technique, two different phases are considered: (i) an offline (computational expensive) phase where all the separated approximations are computed, and (ii) an online phase where the generalized solution is particularized at any desired parameter value (i.e. the online solution). It is important to observe that in contrast to classical a posteriori approaches, like POD [23–25] or reduced basis methods [26,11,12], the online phase with PGD only requires to evaluate a linear combination of the separated representation. Thus, the online phase does not need any new solve and the PGD approximation can be computed in real-time for any parameter value.

In the present framework, the separated functions are particularized for a range of intermediate frequencies and incoming wave directions. This also contrasts with previous works in this field; see for instance [27], where an a posteriori reduced model for the homogeneous Helmholtz equation has to be constructed for each intermediate value.

Moreover, the PGD requires neither to precompute any trial solution (i.e. a snapshot) nor to solve a singular value decomposition problem. Note that this last point can even preclude the application of POD-based techniques for the Helmholtz problem where dense spatial discretizations may be necessary.

The contributions in this paper are applicable to any heterogeneous and unbounded problem governed by the Helmholtz equation, and requiring a large number of evaluations of the input data. However the presented work is inspired from an engineering application: the prediction of water agitation inside harbors. Particularly, two harbors located in the Northeast of Spain are used as test cases. Note that the parameterized wave propagation problem becomes in this case 4D: two spatial coordinates, one for frequency and one for the incoming wave direction. Separated