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Time-integration for ALE simulations of Fluid–Structure Interaction problems: Stepsize and order selection based on the BDF

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Abstract

We present an adaptive algorithm for time integration of fluid–structure integration problems. The method relies on a fully coupled procedure to solve FSI problems in which a naturally GCL-compliant ALE formulation for the finite-element spatial discretization is used. The main originality of the proposed solution procedure is that time integration is performed using automatic order and stepsize selections (hp-adaptivity) based on the Backward Differentiation Formulas (BDF). The stepsize selection is based on a local error estimate, an error controller and a step rejection mechanism. It guarantees that the solution precision is within the user targeted tolerance. The order selection is based on a stability test and a quarantine mechanism. The selection is performed to ensure that no other methods within the family of 0-stable BDF methods would produce a solution of the targeted precision for a larger stepsize (and thus a lower computational time). To improve efficiency, the time integration procedure also relies on a modified Newton method and a predictor. The time adaptive algorithm behaviors and performances are assessed on the vortex-induced translational and rotational vibrations of a square cylinder and on the wake-induced vibrations of 3 cylinders in an in-line arrangement. The algorithm yields substantial CPU time savings (compared to constant stepsize and order integration) while delivering solutions of prescribed accuracies.

Keywords: Time integration; Adaptivity; Fluid–Structure Interaction; Arbitrary Eulerian Lagrangian; Vortex induced vibration; Wake induced vibration

1. Introduction

Fluid–Structure Interaction (FSI) phenomena occur in many natural and man-made environments. A better understanding of these phenomena will lead to engineering improvements such as reducing cost, improving performance or reducing failures. The advancement of computational capabilities has provided an effective means to gain insights into these complex engineering problems. However, for FSI problems, the flow PDEs are coupled with the equations modeling the rigid-body motion of several bodies or the deformation of flexible structures. Hence,

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the spectral content of solutions is usually very rich even at low Reynolds numbers. Furthermore, the simulation of such systems is computationally intensive because the set of PDEs must be integrated over large time intervals since the dynamic of moving bodies or deformable structures interacting with a flow may be long to establish. This means that the time integration of these systems requires an efficient time-stepping procedure to accurately capture the complex time evolution of solutions. Most of the numerical methods presented in the literature rely on low-order time integrators (order 1 or 2) because they are unconditionally stable and relatively easy to implement. However, these low-order time-stepping methods are computationally inefficient when accurate solutions are sought. Furthermore, any time-stepping procedure that uses a constant stepsize will perform poorly if the solution varies rapidly in some parts of the integration interval and slowly in others. This is often the case for FSI applications due to the complex dynamic interaction between the flow and the structure. Also, it is usually not an easy task to predict *a priori* the required stepsize and order to achieve a given targeted accuracy. When not automated, this is inherently a trial-and-error process. This is a sensitive issue in FSI applications for which a small change in the reduced velocity may lead to a very different solution time signature [1]. This paper presents an hp-adaptive time integration procedure for FSI problems. The adaptive algorithm performs both stepsize and order selection to control respectively the solution accuracy and the computational efficiency of the time integration process.

One of the main difficulties associated with the numerical simulation of FSI problems is that the flow equations must be solved on moving grids to account for rigid body motions or flexible structure deformations. Arbitrary Lagrangian Eulerian (ALE) formulations are very popular for solving differential equations on deforming domains [2]. The use of the so-called ALE mapping ensures that the solution evolves along trajectories that are contained in the computational domain at all times. The time deformation of the boundary is predicted along with the solutions. Developing ALE formulations has been a subject of active research. A general overview is provided in Ref. [3] with a particular emphasis on finite-element methods. The bulk of the work focused on the Geometric Conservation Law (GCL) which states that no spurious disturbances should be introduced to a uniform flow due to grid movement effects. However, while this is a sufficient condition for consistency, the GCL does not provide any guaranties or conditions for which the fixed-mesh orders of accuracy of time-integrators would be preserved when used on moving meshes [4]. Hence, the time accuracy of GCL compliant ALE schemes may be (and usually is) reduced from pth order on a fixed grid down to as low as 1st order on a moving grid. In Ref. [3] we presented an ALE approach based on the finite-element discretization to solve 2D/3D viscous flow problems on deforming grids. Our formulation offers the following advantages over the existing ones in the literature:

1. it applies to any finite-element discretization;
2. it applies to problems in any space dimension (*i.e.* $d = 1, 2$ or 3);
3. it is consistent for both fixed and moving grid problems;
4. time-stepping schemes are unchanged and can be of higher-order.

Time integration was performed by the constant- and (non-adaptive) variable-stepsize Backward Differentiation Formulas (BDF) of order 1 to 5. Numerical experiments showed that the *fixed-grid* orders of convergence of the BDF methods are preserved when applied to deforming grid problems by our finite-element ALE formulation.

Adaptivity is a key process to achieve both reliability and efficiency of discretization/integration methods to numerically solve complex sets of equations. Many adaptive procedures to control the spatial discretization error (mesh adaptation) can be found in the literature. Surprisingly, there is a paucity of work by the engineering community to design such procedures to control the time integration error. This is even more surprising in light of the numerous adaptive algorithms that can be found in the discipline that is concerned with solving ODEs. From them, a set of efficient adaptive solvers have been written (mostly during the 70s and 80s). A review of these along with their performances on a set of test cases is presented in Ref. [5], Section III.7 p. 421. Among them, DASSL developed by Petzold has proven extremely efficient at solving ODEs and DAEs (see Ref. [6] for a detailed presentation). For the flow problem alone, a number of adaptive time integration procedures can be found in the literature (see *e.g.* [7–10]). However, to the best of our knowledge, no time adaptive procedure has been designed for FSI problems.

In Ref. [11] we have presented a procedure based on the BDF to obtain efficient time integration of the incompressible Navier–Stokes equations discretized on fixed grids. In practice, the main advantage of the procedure is that the user only has to select the desired accuracy tolerance for a given problem at hand. It frees him from any further considerations about time integration. Ref. [11] focused on unsteady incompressible viscous flow simulations for which, once space discretization is performed, the resulting time-dependent discrete problem is not a system of