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Robust identification of elastic properties using the Modified Constitutive Relation Error

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Abstract

The present study concerns the identification of linear elastic material properties based on inhomogeneous tests and the use of full-field measurements, often based upon inverse approaches. This study presents the formulation of the so-called Modified Constitutive Relation Error (MCRE) method in the context of elastostatics when dealing with uncertain data. Such an approach addresses the concept of the reliability of information and mainly consists in the partitioning of all the available mechanical quantities into a reliable set and a less reliable one, so as to take into account the measurement uncertainties and the error made on the constitutive equation into the formulation, and then allows to identify the sought material properties. The method is split in two steps: the first one consists in defining admissible mechanical fields from all the theoretical and experimental data, for a fixed set of mechanical properties. This is made by the minimization of a criterion allowing a compromise between the constitutive equation and the measurements adequacy. Then, the second step consists in the identification of the sought material properties and takes the form of minimizing a cost function defined by using the above admissible mechanical fields. A comparison with the Finite Element Model Updating (FEMU) method was performed on some numerical examples where realistic perturbations were added. This comparison showed that the MCRE method is more robust towards perturbations for similar input data. Moreover, the proposed method only deals with the available information and does not need additional hypotheses to calculate the mechanical quantities. Eventually, the method was applied to the identification of the shear modulus of an organic matrix composite from experimental data.

Keywords: Modified constitutive relation error; Full-field measurements; Inverse problem; Identification; Measurement perturbations; Elasticity

1. Introduction

The high development of the digital images correlation (DIC) techniques [1–3] based on the exploitation of full-field measurements constitutes one of the main breakthroughs of the last two decades and allows a wise utilization of heterogeneous tests. The latter are needed to characterize material behavior under actual working conditions. Such

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realistic characterization is hardly possible with classic measurement devices, such as the extensometric gauges providing a limited number of measurement points. Among the various mechanical applications, the intensive use of complex composite materials in the aeronautic industry requires to precisely know the parameters governing the mechanical constitutive behavior. Thus, it is necessary to develop identification methods adapted to the richness of full-field measurements and based upon inverse approaches. Indeed, when dealing with heterogeneous strain fields, it is impossible to establish the relation between measurements and stresses using a direct approach.

Otherwise, measurement perturbations are inherent to the experimental tests and affect the accuracy of the identified properties, especially in the elastic range where the magnitude of strains is relatively low. It is then important to take them into account and use a method which remains robust with respect to perturbations. Numerous identification methods were adapted or fully dedicated to the use of full-field measurements. A review of the most widespread methods (Finite Element Model Updating (FEMU) [4–6], Virtual Fields Method (VFM) [7], Equilibrium Gap Method (EGM) [8–10], Reciprocity Gap Method (RGM) [11] and Constitutive Relation Error (CRE) [12,13]) is available in [14,15]. In spite of their advances, some questions impacting the identification accuracy still need attention.

Actually, two principal shortcomings can be noted about the above identification methods. First, most of them perform the identification only over the measurement zone. Since it is difficult to obtain a measurement zone covering the whole surface of the studied specimen, a calculation performed on such a restricted zone may cause the loss of any information available on the border of the specimen. It can consist in free-edge information considered as reliable, or the measurement of the global load when applied to a larger zone than the measurement zone. Second, for standard identification methods like FEMU, boundary conditions are required. Consequently, for a calculation made in the measurement zone, a standard way to define boundary conditions is to deal with perturbed Dirichlet boundary conditions (extracted from the measured full-field). Otherwise, for a larger calculation zone, the definition of a set of boundary conditions often requires additional hypotheses. For both cases, either measurement or modeling errors lead to artifacts into the identification algorithm.

The challenge of the present study is to propose a method dedicated to full-field measurements which allows a calculation not limited to the measurement zone, and with no additional hypotheses on the unavailable information. Furthermore, the proposed method considers the whole theoretical and experimental information (but not more), and takes into account the reliability of the measurements, making the identification robust towards errors. The so-called Modified Constitutive Relation Error (MCRE) was used at first for model updating in vibration dynamics within the framework of modal analysis in [16], then adapted to forced vibration problem in [12]. The MCRE was coupled to a model reduction technique and applied to abundant experimental data in [17], then in [18] for the characterization of acoustic absorbers. The method was adapted to the exploitation of full-field measurements in [19] where boundary conditions were known on the whole boundary. It was applied to the identification of elastic properties within the framework of transient dynamics in [13] where the MCRE turns out as a robust method towards measurement perturbations. The extension of the identification to non linear cases was taken on for viscoplastic properties and rupture parameters of composite materials in [20]. More recently, the MCRE method was applied within a dynamic frequency analysis in order to identify elastic properties in [21]. Most of the works on DIC with the constitutive relation error use the standard CRE which requires strain measurements. When using classic DIC, it implies the differentiation of the displacement measurements leading to the increase of the perturbations level and the use of a specific strain reconstructor that filters the noise (as proposed in [22]). According to [23], such a non-mechanical filtering step would lead to a bias in the identification results (even though it should be quantified for the CRE approach). By using the displacements in the MCRE approach, we avoid any non-mechanical filtering.

The elastostatic problem addressed in this study is solved through a sequential minimization of a functional composed of two quadratic terms: one related to the error made on the constitutive relation and the other one to the gap between simulated and experimental data. The first minimization, namely the basic problem, leads to the construction of admissible kinematic and static fields optimal with respect to the available information, when the second, namely the identification problem, yields the sought material properties using the optimal admissible fields.

The paper is organized as follows: the inverse problem and the development of the corresponding formulation of the MCRE method are presented in Section 2. The numerical implementation and the solving algorithm are then presented is Section 3. In Section 4, the proposed method is applied to some numerical cases to highlight its robustness towards perturbations, and a comparison between the MCRE and the FEMU methods is performed. Finally, the identification of a homogeneous shear modulus is performed for a composite material using experimental data.