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A multi-dimensional transfer function approach to photo-acoustic signal analysis

Natalie Baddour*

Department of Mechanical Engineering, University of Ottawa, 161 Louis Pasteur, Ottawa, Ontario, Canada K1N 6N5

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Abstract

Photo-acoustic signal generation has shown potential for medical tomography. This paper aims to present a consistent and unified approach to the mathematical modelling of the photo-acoustic problem, using a transfer function approach. A generalized version of the Fourier slice theorem is presented and proved. Reconstruction algorithms can be developed based on specific cases of this general theorem. Closed-form solutions to special cases are given in Cartesian, cylindrical and spherical polar coordinates. These can be used to simulate the forward problem and as test cases for any reconstruction algorithms.

Keywords: Modelling; Photo-acoustic; Transfer function; Fourier; Imaging

1. Introduction

Photo-acoustic signal generation is a new technique, which has demonstrated great potential for non-invasive medical tomography. With this technique, a short-pulsed laser source is used to irradiate the sample. The energy absorbed produces a small temperature rise, which induces a pressure inside the sample through thermal expansion. This pressure acts as an acoustic source and generates further acoustic waves, which can be detected by ultrasound transducers positioned outside the sample. Since there is a large difference in optical absorption between blood and surrounding tissue, the laser irradiation induces an

*Tel.: +1 613 562 5800; fax: +1 613 562 5177.
E-mail address: nbaddour@uottawa.ca

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ultrasound wave on the inhomogeneities within the investigated volume. Hence, the acquired photothermo-acoustic signals carry information about the optical absorption property of the tissue. This approach is thus suitable for the imaging of the micro-vascular system or for tissue characterization, with contrast similar to that of pure optical imaging and spatial resolution similar to that of pure ultrasonic imaging. It therefore combines the advantages of two imaging modalities in a single modality. The issue of the strong scattering of light in media like biological tissue is overcome and the ability of acoustic waves to travel long distances without significant distortion or attenuation is also exploited. Photo-acoustic detection has shown concrete promise of imaging in turbid media at depths potentially through the full thickness of skin [1,2].

Several groups of researchers have made significant progress towards achieving these imaging goals [2–6]. However, a number of different approaches have been developed by each group. This paper thus aims to present a unifying framework for the mathematical theory of photo-acoustic imaging, using a transfer function/Green's function approach. A photo-acoustic Fourier theorem is presented and proved. Related reconstruction results are given in [4,7,8], which can be shown to be special cases of the general results shown here. Those papers do not use multi-dimensional Fourier transforms and thus cannot demonstrate that these results can all be interpreted in a concise way as specific cases of a generalized Fourier theorem. This paper aims to present the theorem in a clear and general way, with no assumption regarding the detection geometry. This theorem is extremely important as it forms the basis for most reconstruction algorithms. The representations in this paper are embedded in time-dependent, spatially three-dimensional (3D) descriptions. Different coordinate systems are considered.

Section 2 presents the governing equations for this problem. Section 3 considered the solution in terms of Cartesian coordinates and presents several special cases, which can be solved in closed form and which are useful for elucidating the main features of the proposed approach. Section 4 formally introduces the Green's function, transfer function approach to the problem and Section 5 derives the relevant functions in spherical polar coordinates. Section 6 develops the photo-acoustic Fourier theorem and is the main result of this paper. Section 7 considers the application of this theorem to the special case of spherically symmetric heterogeneity functions. Section 8 discusses the disadvantage of using short Dirac–delta pulses as input pulses and Section 9 considers the development of the proposed approach in terms of cylindrical polar coordinates and presents an example. Section 10 summarizes the main mathematical results that have been developed in the paper and Section 11 concludes the paper.

2. Governing equations

The physical principle behind this imaging modality is the photo-acoustic effect. This entails the generation of an acoustic wave as a result of the absorption of light pulse. While optical energy can be converted to mechanical energy through various pathways, it is often the case that thermal expansion is the dominant mechanism. In pulsed photo-acoustic tomography, the pulse duration is so short that the thermal conduction time is greater than the thermo-acoustic transit time and the effect of thermal conduction can be ignored [1]. The equation describing the thermo-acoustic wave propagation with a thermal expansion source term is given by [1,2,6,7,9]

$$\nabla^2 p(\vec{r},t) - \frac{1}{c_s^2}\frac{\partial^2}{\partial t^2} p(\vec{r},t) = -\frac{\beta_s}{C_p}\frac{\partial}{\partial t} H(\vec{r},t) \tag{1}$$