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ScienceDirect
Journal of the Franklin Institute 345 (2008) 926–941
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

Design of sliding surface for mismatched uncertain systems to achieve asymptotical stability

Chih-Chin Wen, Chih-Chiang Cheng*

Department of Electrical Engineering, National Sun Yat-Sen University, Kaohsiung 804, Taiwan, ROC

Received 16 June 2005; received in revised form 26 November 2007; accepted 4 June 2008

Abstract

The design of an adaptive sliding mode control (SMC) scheme is proposed in this paper for stabilizing a class of dynamic systems with matched and mismatched perturbations. Two methods for designing a novel sliding surface function are introduced first. By utilizing a pseudocontrol input in the sliding surface function, one cannot only suppress the mismatched perturbations in the sliding mode, but also obtain the property of asymptotical stability. Then a sliding mode controller is designed to drive the controlled systems to the designated sliding surface in a finite time. Adaptive mechanism is also embedded in the controller as well as in the sliding surface function designed from the second method to overcome the perturbations, so that the informations of upper bound of perturbations are not required. An application of flight control and experimental results of controlling a servomotor are also given for demonstrating the applicability of the proposed control scheme.

Keywords: Sliding mode control; Mismatched uncertain systems; Adaptive control; Asymptotical stability

1. Introduction

According to the space spanned by the input gain matrix, one can divide the perturbations of systems to be controlled into two parts, one is matched part, and the other is mismatched part [1]. It is well known that if perturbations of the system satisfy matching condition, then the behavior of controlled system is insensitive to this matched perturbations in the sliding mode when employing SMC technique. But if the mismatched

*Corresponding author. Tel.: +886 7 5252000x4133; fax: +886 7 5254199.
E-mail address: chengcc@mail.ee.nsysu.edu.tw (C.-C. Cheng).

0016-0032/$32.00
doi:10.1016/j.jfranklin.2008.06.003

perturbations exist, then the dynamics of systems will still be affected by these mismatched perturbations even when system is in the sliding mode.

Many scholars have focused their researches on the designing of SMC systems with mismatched perturbations. One typical achievement of these researches is that the state trajectories of the controlled systems can be driven into a bounded region so that the stability of the controlled system is ensured. These researches include [2–7].

It is a more difficult job to design sliding mode controllers for systems with mismatched perturbations in order to achieve the property of asymptotical stability. When designing these control systems, most of the researches have to put some restrictions on the perturbations that the systems might encounter, and also need the information of upper bounds of these perturbations. Kwan [8] treated certain states as inputs to a reduced-order system and designed fictitious controllers for these inputs, which can tackle the system's mismatched uncertainties, but these mismatched model uncertainties should be in the range space of certain matrix of the nominal system. The technique of LMI is used to achieve quadratical stability in [9–11], where the model uncertainties have to be in a special factorized form, i.e., $\Delta\mathbf{A}(t,\mathbf{x}) = \mathbf{D}\mathbf{F}(t,\mathbf{x})\mathbf{E}$, $\mathbf{D}$ and $\mathbf{E}$ are two known constant matrices, and the unknown matrix $\mathbf{F}(t,\mathbf{x})$ has to satisfy $\|\mathbf{F}(t,\mathbf{x})\| \leqslant 1$. Chan et al. [12], Kim et al. [13] employed SMC to deal with mismatched uncertainties by making similar assumptions for uncertainties as in [9–11]. By introducing two sets of switching surfaces for each subsystem, Tsai et al. [14] restricted the rank of mismatched uncertainties matrices to achieve asymptotical stability for a class of large-scale systems. Dynamic output feedback controllers are proposed in [15] for mismatched uncertain variable structure systems, where the upperbound of mismatched uncertainties has to be smaller than a designed constant to achieve asymptotical stability in the sliding mode. A nonlinear integral-type sliding surface function is designed in [16] for both matched and mismatched uncertain systems, and the stability conditions are also analyzed for systems with mismatched uncertainties.

It is observed that all the aforementioned works required the information of upper bounds of perturbations that the control systems might encounter. In fact, due to the complexity of the structure of the controlled systems and perturbations, it is in general difficult to obtain or too expensive to assess these information in many practical applications. Therefore, a strategy in which the boundary values of the perturbations can be easily obtained is required. The adaptation method proposed in [17–19] offers a simple and effective tool to solve this problem.

Based on the Lyapunov stability theorem and SMC strategy, in this paper we propose two methods to design a novel sliding surface for stabilizing a class of perturbed dynamic systems. The first method is for systems with $m \leqslant n$, that is, the number of input channel is smaller or equal to the dimension of the system. Although the information of the upperbound of mismatched perturbation is still required in this case, it is quite easy to modify this sliding surface design for the systems with $n \leqslant 2m$ (the second method), so that the information of the upperbound of perturbation is not required. Due to this novel sliding surface design, one can suppress the influence of the mismatched perturbations on the system's dynamics when system is in the sliding mode, and guarantee the property of asymptotical stability. Another advantage of this approach is that the restrictions of the mismatched perturbations that the system can bear are substantially relaxed. Finally an application of flight control and experimental results of controlling a servomotor are demonstrated for showing the feasibility of the proposed control scheme.