Available online at www.sciencedirect.com
ScienceDirect
Journal of the Franklin Institute 345 (2008) 226–232
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

Biological applications of the "Filtered" Van der Pol oscillator

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Received 9 August 2005; accepted 23 August 2007

Abstract

The present article deals with the development of an oscillatory model, which generates waveforms corresponding to ECG patterns. The present oscillatory system relies on coupling of oscillators derived from the famous VDP oscillator. We demonstrate that inducing a relaxation type of dynamics in the models contributes to their successful generation of ECG like signals. Furthermore, an interesting affinity is found, which associates the present models with a version of the well-known practical Wien Bridge oscillator. The presently discussed system relies on coupled elementary oscillator units. The present coupling is due to merely two units. The model, however, is likely to become even more realistic by coupling in the same manner an assembly of relatively many oscillators.

Keywords: Heart dynamics; Nonlinear; Van der Pol; Coupled oscillators; Wien Bridge oscillator

1. Introduction

We have recently been involved in an investigation related to the participation of Mg ions in determining the heart electrical activity and influencing its pace making mechanism. These ions are regarded as affecting the electrical coupling between the heart muscle cells.

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0016-0032/$32.00
doi:10.1016/j.jfranklin.2007.08.005

Hence, they should affect the heart mechanical dynamics as well as the heart electrical activity, which is reflected through the ECG signals [1]. We have in addition accidentally observed that certain relaxation oscillators derived from the family of Van der Pol related models generate signals strongly reminding ECG patterns. Furthermore, an appropriate coupling of two such oscillators leads to the generation of signals, which appear even more closely related to ECG signals. It is, therefore, assumed that the coupling mechanism affecting the latter relaxation oscillators may bear some similarity to the effect of the Mg ions and could be helpful in developing models for describing the influence of Mg ions. A known effort aiming at similar objectives was published [2]. Their model, however, which represents merely one of the coupled oscillators is already of the fifth order, while the presently suggested oscillator is of second order only. The coupling, however, in the present case can be regarded somewhat complicated when judged from system theorists' viewpoint, since a delay is utilized for this purpose. But this does not cause the application of the system for simulation purposes to be involved. In addition, the present work besides its possible contribution to biomedical engineering possesses a tutorial value in reviewing various relaxation oscillators as well as oscillators related to the Van der Pol family. A relationship has even been found to be a derivative of the practical Wien Bridge oscillator.

2. Relatively simple Van der Pol related oscillators

The oscillator model dealt with in the present work can be regarded as a relaxation oscillator strongly related to the Van der Pol equation:

$$\begin{aligned} \dot{x} &= y + \varepsilon(1 - \mu y^2)x, \\ \dot{y} &= -x, \end{aligned} \tag{1}$$

where μ is a positive parameter. This Van der Pol equation [3] is one of the simplest capable of compactly representing the various features of practical oscillators behavior. It represents, when $\varepsilon \to 0$, a purely basic oscillatory dynamics. It then becomes a simply conservative harmonic oscillator. The additional term, whose coefficient is ε, is a nonlinear damping term responsible for stabilizing the oscillator amplitude (it acts to shape a limit cycle). The stable limit cycle related behavior occurs when $\varepsilon > 0$ and $\mu > 0$. The stabilized behavior is due to the fact that, when the amplitude is large, y^2 is on the average large, and it causes the damping (the damping term is the one associated with ε) to be on the average negative, which leads the amplitude to decrease. When, on the other hand, the amplitude is small, the damping coefficient is positive, which causes the amplitude to grow. These processes balance themselves and the amplitude is maintained constant in steady state. The oscillator is then considered to have reached a "limit cycle". The oscillator waveform becomes more and more distorted with an increase in ε. It even demonstrates a so-called relaxation type of oscillations when ε is increased even further. The relaxation oscillation in the latter case is characterized by the oscillator generating a train of short symmetrical (symmetrical in the sense of having a repetitive train, where a positive impulse is always followed by an inverted impulse of the same shape) equi-distant impulses. The periods between the impulses are known as relaxation periods, where the oscillator output is relatively small (the oscillator relaxes). The latter waveforms descriptions are due to the