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Short communication

Non-iterative pole placement technique: A step further

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Abstract

This paper comes back to the hard problem of pole placement by static output feedback: let a triplet of matrices $\{A; B; C\} \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m} \times \mathbb{R}^{p\times n}$ be given, find a matrix $K \in \mathbb{R}^{m\times p}$ such that the spectrum of $A + BKC$ equals a specified set. More precisely, this article focuses on the derivation of *non-iterative* techniques, based upon the notion of eigenstructure assignment, to solve the problem, especially when Kimura's condition does not hold. Some solutions can sometimes be found.

Keywords: Pole placement; Eigenstructure assignment; Kimura's condition; Output feedback

1. Introduction

Since the contributions of Davison [1–3], the static output feedback pole placement problem is one of the most investigated problems in the control community and is actually not yet completely solved. For interesting surveys about that problem, and more generally, about static output feedback, see [4–6]. The problem consists in considering matrices $\{A; B; C\} \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m} \times \mathbb{R}^{p\times n}$ and in finding a matrix $K \in \mathbb{R}^{m\times p}$ such that the spectrum of the closed-loop matrix $A_c = A + BKC$ equals a set of arbitrarily specified values $\{\lambda_i, i = 1, \ldots, n\}$. A first question is the existence of a solution. This question is in itself a hard topic. Various crucial contributions are recalled in [7]. In Brockett [8], it is proven that the generic condition for pole placement is $mp \geqslant n$ provided that the problem is solved in the field of complex matrices. In the field of real matrices (corresponding to the actual

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problem), a sufficient condition for generic pole assignability was proposed by Wang [9] with alternative proofs in [10–12]: $mp > n$ (only generic as illustrated by the counterexample of [13]). Although those results are fundamental to understand the present control problem, another challenge is to derive methods that exhibit a solution. The only one that is available for all the cases encompassed by $mp > n$ is presented in [9]. It relies on an optimization process whose parameters may be difficult to tune. The basic reason might be that the static output feedback pole placement problem is NP-hard [14].

Regarding this result of NP-hardness, it becomes illusory to always expect to derive a direct technique (by "direct", the authors mean non-iterative) that solves the problem provided that Wang's condition holds. Nevertheless, it is important to list the various cases for which one can generically derive a suitable static feedback control law through direct methods. There are several approaches to solve the problem. Among those approaches, the authors focus on those based upon the notion of eigenstructure assignment, that is those inspired from the work of Moore [15] on the state feedback. When a static output feedback is to be computed, the best results are the so-called geometric approach [16], the parametric approach [17] and the approach based on two Sylvester equations coupled by an orthogonality condition [18]. All these methods require that Kimura's condition ($m + p > n$) [19] be satisfied. Indeed, this condition has been considered as the best sufficient condition for generic assignability for a long time [8,19–21], until the work of Wang. From a practical point of view, it remains an interesting frontier between the systems for which a solution can be easily derived and those for which a solution exists but is not easily reached. A significant attempt to find a solution when Kimura's condition does not hold is [22]. In this contribution, an optimization procedure may allow to solve the case $m + p = n$ (but some convenient initialization has to be guessed). Interesting insights can also be found in [23]. Recently it has been shown that the case $m + p = n < mp$ could generically be solved by a direct method [24]. It must be mentioned that, as in [24], only the case of eigenvalues with multiplicity one is here addressed.

The present paper aims at highlighting some other cases that allow to find a solution by a direct computation, even if Kimura's condition is not verified. For clarity, the next three sections recall the content of [24] in which the approach used is a basis for the present work. Indeed, Section 2 states the problem and recalls some classical properties of the eigenstructure. In Section 3, the spectrum assignment is characterized by an "assignment equation". In Section 4, assignment techniques are deduced for the above-mentioned cases: complete placement under Kimura's condition, partial placement under $mp > m + p$, complete placement under $m + p = n < mp$. Section 5 is the actual contribution of the present paper. It shows how the work of [24] can be carried on to encompass some cases for which $m + p < n < mp$. Section 6 proposes some numerical illustration before the paper is concluded in Section 7.

Throughout the paper, $\bar{M}$ is the conjugate of M and M' is its transpose (also conjugate if M is complex). I is the identity matrix and 0 is the null matrix of appropriate dimensions. Span(M) is the subspace image of the linear application associated to M whereas Ker(M) is the right orthogonal complement (right nullspace) of this application. $\lambda(A)$ is the spectrum of A. Other notations are usual.

2. Problem statement and background

Before to state the problem, the next preliminary definition is introduced.