Available online at www.sciencedirect.com
ScienceDirect
Journal of the Franklin Institute 347 (2010) 452–466
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

# The solution of the Bagley–Torvik equation with the generalized Taylor collocation method

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Received 4 December 2008; received in revised form 10 March 2009; accepted 12 October 2009

**Abstract**

In this paper, the Bagley–Torvik equation, which has an important role in fractional calculus, is solved by generalizing the Taylor collocation method. The proposed method has a new algorithm for solving fractional differential equations. This new method has many advantages over variety of numerical approximations for solving fractional differential equations. To assess the effectiveness and preciseness of the method, results are compared with other numerical approaches. Since the Bagley–Torvik equation represents a general form of the fractional problems, its solution can give many ideas about the solution of similar problems in fractional differential equations.


*Keywords:* Bagley–Torvik equation; Generalized Taylor collocation method; Adomian decomposition method; Differential transform method

## 1. Introduction

In fractional calculus, the Bagley–Torvik equation

$$Ay''(x) + By^{(3/2)}(x) + Cy(x) = f(x) \tag{1.1}$$

($A \neq 0$ and $B$, $C \in R$) has an outstanding role for being a model of motion of a rigid plate immersed in a Newtonian fluid. This equation is first proposed by the authors of [1], and then many authors have analyzed this equation by different approximate numerical methods. Podlubny [2] has also investigated the solution of this problem and proposed a numerical method in his book. He also gave the analytical solution of the Bagley–Torvik

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0016-0032/$32.00 
doi:10.1016/j.jfranklin.2009.10.007

equation with homogeneous initial conditions by using Green's function. But, in practice, these equations cannot be evaluated easily for different functions $f(x)$. Trinks and Ruge [3] modeled the Bagley–Torvik equation again and compared the numerical solution by using an alternative time discretization scheme with the Podlubny's numerical solution. Leszczynski and Ciesielski [4] proposed a numerical solution of Bagley–Torvik equation considering the equation as a system of ordinary differential equations using the Abel integral equations. Edwards et al. [5] solved linear multiterm fractional differential equations as a system. So, they solved the Bagley–Torvik equation by using the reduction of the problem to a system. In a similar manner El-Sayed et al. [6] solved the Bagley–Torvik equation of arbitrary order by considering the equation as a system. Another method proposed for the solution of the fractional differential equations is Adomian decomposition method (ADM). Ray and Bera [7] applied Adomian decomposition method for the solution of Bagley–Torvik equation and obtain the same solution as the Podlubny's solution by Green's function. In this problem, $f(x)$ is considered as Heaviside function. In a similar study, Hu et al. [8] got the solution of the Bagley–Torvik equation by using Green's function. Daftardar-Gejji and Jafari [9] discussed the solution of the Bagley–Torvik equation as a system of differential equations by the Adomian method. Arikoglu and Ozkol [10] applied differential transform method (DTM) to Bagley–Torvik equation for specified initial conditions and a certain function $f(x)$. Çenesiz [11] proposed differential transform method using the grid points for the solution of the Bagley–Torvik equation. He also showed the agreement of his solution with Podlubny's solution.

In this paper, we present a new generalization of the Taylor collocation method that will extend the application of the method to differential equations of fractional order. The new technique can be called as generalized Taylor collocation method (GTCM) and is based on the Taylor collocation method [25,26], generalized Taylor's formula [30] and Caputo fractional derivative [27–29]. Using the collocation points, the GTCM transforms the given fractional differential equation and initial conditions to matrix equation including unknown generalized Taylor coefficients. By means of the matrix equation and the computer package program Maple 11, the generalized Taylor coefficients can be computed.

## 2. Basic definitions

In this section we present some basic definitions and important properties of fractional calculus [12–20].

**Definition 1.** A real function $f(x)$, $x<0$, is said to be in the space $C_\mu$, $\mu \in R$, if there exists a real number $p<\mu$, such that $f(x) = x^p f_1(x)$, where $f_1(x) \in C(0,\infty)$, and it is said to be in the space $C_\mu^n$ if $f^{(n)} \in C_\mu$, $n \in N$.

**Definition 2.** The fractional integral operator ${}_aJ_x^\alpha$ (Riemann–Liouville operator) of order $\alpha \geq 0$ of a function $f \in C_\mu$, $\mu \geq -1$, is defined by

$$
\begin{aligned}
{}_aJ_x^\alpha f(x) &= \frac{1}{\Gamma(\alpha)} \int_a^x (x-u)^{\alpha-1} f(u)\, du, \quad x \geq a \\
{}_aJ_x^0 f(x) &= f(x)
\end{aligned}
\tag{2.1}
$$