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Journal of the Franklin Institute 347 (2010) 467–482

Journal of The Franklin Institute

www.elsevier.com/locate/jfranklin

Maximum principle for optimal boundary control of the Kuramoto–Sivashinsky equation

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Received 29 November 2007; received in revised form 20 September 2009; accepted 12 October 2009

Abstract

In this paper, the optimal boundary control for the Kuramoto–Sivashinsky equation is considered. The Dubovitskii and Milyutin functional analytical approach is adopted in investigation of Pontryagin's maximum principles of the system in both fixed and free final horizon cases. The necessary conditions are, respectively, presented for the optimal boundary control problems in these two cases.

MSC: primary 49K20; 35Qxx

Keywords: Kuramoto–Sivashinsky equation; Optimal boundary control; Maximum principle; Necessary condition

1. Introduction

The Kuramoto–Sivashinsky equation (KS equation for short)

$$u_t + \nabla^4 u + \nabla^2 u + \tfrac{1}{2}|\nabla u|^2 = 0, \tag{1.1}$$

where ∇^2 denotes the Laplacian, ∇^4 the biharmonic operator, and ∇ the gradient, is a fourth-order nonlinear partial differential equation ([34]) which can be used to describe many important physical and chemical systems. In the 1970s, the KS equation was introduced by Kuramoto [17] in studying instabilities on interfaces and flame fronts (see also [18,19]), and by Sivashinsky [29] for the investigation of phase turbulence in chemical

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0016-0032/\$32.00
doi:10.1016/j.jfranklin.2009.10.008

oscillations. Since then, it receives considerable attention due to its importance in the chemical engineering and mathematical physics. Most of the investigations center on the well-posedness and dynamics of the equation. For example, by the KS equation, Matar et al. [24] study the nonlinear stability and dynamic behavior of falling fluid films; Annunziato et al. [2] consider the unstable flame front propagation in uniform mixtures; and Ramaswamy et al. [27] analyze the interfacial instabilities in thin-film flows. For other interesting results, refer to [4,9,28,31,33], name just a few.

Nonetheless, when the KS equation is involved in control issues, the situation turns to be completely different and there are few known researches yet. Just like Liu and Krstić [21] said, in this respect, control problems for the KS-equation are largely unexplored and need more attention. Furthermore, among the known works are mostly on the feedback control of the said equation. For instance, Liu and Krstić [21] address the problem of Dirichlet and Neumann boundary control of the KS equation and develop a Neumann feedback law that guarantees L^2-global exponential stability and H^2-global asymptotic stability for small values of the anti-diffusion parameter; in [20] Lee and Tran adopt the reduced-order methods to obtain a reduced-order system from which the feedback controller can be designed and synthesized; using the stochastic KS equation, Lou and Christofides [22] investigate the feedback control problem of surface roughness in sputtering processes; Kobayashi [16] considers the adaptive stabilization of the KS equation and under the existence of bounded deterministic disturbances the adaptive stabilizer is constructed; Lou and Christofides [23] compute the optimal actuator/sensor location problem for the stabilization via nonlinear static output feedback control; Armaou and Christofides [3] synthesize linear finite-dimensional output feedback controllers to achieve stabilization of the zero solution; moreover, Christofides and Armaou [8] study the problem of global exponential stabilization of the one-dimensional KS equation via distributed static output feedback control and show that the designed scheme stabilizes the KS equation.

This paper is concerned with the optimal boundary control investigation of the KS equation. On this topic, much fewer results are known to the best of our knowledge. Hu and Temam [15] consider the robust boundary control problem for the KS equation and prove the existence and uniqueness for the robust control problem; He et al. [13] study numerical aspects of controllability and optimal control of the KS equation employing the distributed control and periodic boundary conditions. Recently, an interesting application of the KS equation is to describe the control problem of ripple formation in abrasive water-jet cutting (see [11,26]). By the generalized KS equation, Maurer and Theißen [25] compute the optimal distributive control for the water-jet cutting model.

Indeed, the feedback control of dynamical systems has many merits comparing to the open-loop control. However, an undeniable fact is that the latter, the open-loop control has its own advantages in the investigations of the infinite dimensional system, such as the efficiency and accuracy of the open-loop control algorithms as well as the robustness aspect of investigational systems ([30]). Just as Ho and Pepyne [14] said in "The No Free Lunch Theorem of Optimization (NFLT)", a general-purpose universal optimization strategy is impossible. Therefore the open-loop control investigation to the KS equation is both necessary and interesting.

In this paper, the optimal boundary control problems of the KS equation are considered in both fixed and free final horizon cases. By the Dubovitskii and Milyutin functional