Available online at www.sciencedirect.com
ScienceDirect
Journal of the Franklin Institute 345 (2008) 437–451
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

Strongly absolute stability problem of descriptor systems: Circle criterion

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Received 28 August 2006; accepted 12 December 2007

Abstract

This paper considers a Lur'e-type descriptor system (LDS) which is a feedback system whose forward path is a single-input and single-output linear time-invariant descriptor system and feedback path is an uncertain nonlinearity with sector constraint. The notion of strongly absolute stability is defined for LDS and some sufficient conditions are presented. If the forward path of LDS is impulsive free, the classical circle criterion (CC) for Lur'e-type standard systems (LSS) is shown to be still valid for LDS. If the forward path of LDS is not impulsive free, a new CC is derived by using a modified Nyquist stability condition. Finally, numerical examples are given to illustrate the effectiveness of the obtained results.

Keywords: Lur'e-type systems; Descriptor systems; Strongly absolute stability; Nyquist stability condition; Circle criterion

1. Introduction

In the last two decades, descriptor systems have been one of the major research fields of control theory due to their comprehensive applications in the Leontief dynamic model [1], electrical and mechanical models [2,3], etc. Depending on the applicable areas, these models are also called singular systems, semi-state systems, differential–algebraic systems, or generalized state-space systems. There have been many results on the stability problem

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0016-0032/$32.00
doi:10.1016/j.jfranklin.2007.12.002

of descriptor systems. Lewis [4], Dai [5], and Ishihara and Terra [6] presented sufficient and necessary conditions for admissibility of linear time-invariant descriptor systems. Vladimir [7,8] and Vladimir and Mirko [9] investigated the stability of nonlinear descriptor systems under an assumption that the set of consistent initial conditions is given. Wu and Mizukami [10] extended the Lyapunov stability theory for standard state-space systems to nonlinear descriptor systems. And in [11], the authors derived a sufficient condition for the system to be locally asymptotically stable. As concluded in [12], there are several difficulties in the study of stability problem for nonlinear descriptor systems: (i) it is not easy to satisfy conditions of the existence and uniqueness of solutions; (ii) there often exist impulses and jumps in the solutions; (iii) it is difficult to calculate the derivatives of Lyapunov functions along the solutions.

In 1944, Lur'e and Postnikov introduced a novel method called ''nonlinearities isolation method'' to deal with stability problem of nonlinear systems. For many practical control systems, by using this method, the nonlinear characteristic can be separated, which results in a feedback system called Lur'e-type system whose forward path is a linear time-invariant system and the feedback path is a nonlinearity with sector constraints [13]. Lur'e-type standard state-space systems (LSS) have been widely investigated and the most celebrated ones are Popov criterion (PC) and circle criterion (CC) [14,15]. PC is less conservative than CC because the Lyapunov function used by PC is a Lur'e-type Lyapunov function which explicitly depends on the nonlinearity, while CC is related to a quadratic Lyapunov function. In addition, CC can deal with more diverse nonlinearities including time-varying ones. However, investigation on Lur'e-type descriptor systems (LDS) is few. In [16], an LMI-based strictly positive real (SPR) lemma is given for discrete-time descriptor systems. Under the admissibility and SPR assumption of the involved linear time-invariant descriptor systems, it shows that the globally asymptotic stability of the feedback connection is guaranteed for the whole class of memoryless time-varying nonlinearities with dynamics constrained in the first and third quadrants. But it does not consider the impulsive behavior of the overall system.

In this paper, we investigate the stability of LDS. First, the notion of index of nonlinear descriptor systems is recalled. For convenience and without any confusion, an index one nonlinear descriptor system is called to be impulsive free in this paper. Consequently, strongly absolute stability of LDS is defined to be globally stable and impulsive free. Such a concept is a generalization of the absolute stability of LSS as well as the admissibility of linear time-invariant descriptor systems. Then, we try to generalize the well-known CC for LSS to LDS. Following the methodologies of absolute stability of LSS, the so-called linearized system of LDS is introduced, by which a necessary condition is given for the strongly absolute stability of LDS. For a representative class of the nonlinear characteristic, a necessary and sufficient condition for LDS to be strongly absolutely stable is derived by which a sufficient condition is given. Such a condition plays an important role in the derivation of the CC for LDS. It is shown that the existing CC for LSS can be used directly if the forward path of LDS is a impulsive free linear time-invariant descriptor system. When the forward path of LDS is not impulsive free, or equivalently, the open-loop transfer function is improper, we derive a new CC by a modified Nyquist criterion that can verify the admissibility of the closed-loop system when the open-loop transfer function is improper. Finally, we present some numerical examples to illustrate the obtained results.