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Numerical instability of deconvolution operation via block pulse functions

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Abstract

This paper characterizes oscillations found in block pulse function (BPF) domain identification of open loop first-order systems with step input. A useful condition for occurrence of such oscillations is presented mathematically. For any positive value of 'ah', oscillations are observed to occur, where h is the width of BPF domain sub-interval and $1/a$ is the time constant of the first-order system under consideration.

Keywords: Block pulse functions; Identification; Oscillation

1. Introduction

Piecewise constant basis functions [1] were introduced by Alfred Haar in 1910. Its square wave nature attracted many researchers and other such functions, e.g., Walsh functions, slant functions, block pulse functions (BPF) [1,2] followed. Among all these functions, the BPF set proved to be the most fundamental [3] and it enjoyed immense popularity in different applications in the area of control systems [4], including analysis and identification [5] problems.

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However, it has been observed that BPF domain identification of a control system, in absence of measurement noise, yields oscillatory results when 'deconvolution' [5] based operational technique is used. It is implied that presence of measurement noise will aggravate the situation further.

In the present work, the reason for such 'instability' has been explored and a mathematical explanation has been provided with suitable numerical examples.

The main objectives of the present work are:

(i) To identify an open loop single-input–single-output (SISO) system in the BPF domain, in absence of measurement noise, using established knowledge and draw attention to undesired oscillations.
(ii) To find mathematical reasons for such undesired oscillations.

2. Brief review of BPF domain convolution and deconvolution process [4,5]

A set of BPF $\mathbf{\Psi_{(m)}(t)}$ containing m-component functions in the semi-open interval $[0, T)$ is given by

$$\mathbf{\Psi_{(m)}(t)} \triangleq [\psi_0(t) \;\; \psi_1(t) \;\; \psi_2(t) \;\; \cdots \;\; \psi_i(t) \;\; \cdots \;\; \psi_{(m-1)}(t)]^{\mathrm{T}}, \tag{1}$$

where $[\cdots]^{\mathrm{T}}$ denotes transpose.

The ith component $\psi_i(t)$ of the BPF vector $\mathbf{\Psi_{(m)}(t)}$ is defined as

$$\psi_i(t) = \begin{cases} 1 & ih \leqslant t < (i+1)h, \\ 0 & \text{elsewhere}, \end{cases}$$

where $h = T/m$ and $i = 0, 1, 2, \ldots, (m-1)$.

A square integrable time function $f(t)$ of Lebesgue measure may be expanded into an m-term BPF series in $t \in [0, T)$ as

$$f(t) = [f_0 \; f_1 \; f_2 \;\; \cdots \;\; f_i \;\; \cdots \;\; f_{(m-1)}]\mathbf{\Psi_{(m)}(t)} = \mathbf{F}^{\mathrm{T}}\mathbf{\Psi_{(m)}(t)}. \tag{2}$$

The constant coefficients f_i's in Eq. (2) are given by

$$f_i = \frac{1}{h}\int_{ih}^{(i+1)h} f(t)\psi_i(t)\,\mathrm{d}t. \tag{3}$$

Consider a SISO time-invariant system, shown in Fig. 1, under zero initial conditions. An input $r(t)$ is applied to the system at $t = 0$. The impulse response of the plant, $g(t)$, is absolutely integrable over $t \in [0, T)$. It follows that each of the time-functions, $r(t)$, $g(t)$ and the output $c(t)$, can be approximated by a block pulse series.

Referring to Fig. 1, the output $c(t)$ is given by the convolution integral

$$c(t) = \int_0^t r(\lambda)g(t-\lambda)\,\mathrm{d}\lambda. \tag{4}$$

Fig. 1. Plant modelled by impulse response function.