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A new continuous action-set learning automaton for function optimization ☆

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Abstract

In this paper, we study an adaptive random search method based on continuous action-set learning automaton for solving stochastic optimization problems in which only the noise-corrupted value of function at any chosen point in the parameter space is available. We first introduce a new continuous action-set learning automaton (CALA) and study its convergence properties. Then we give an algorithm for optimizing an unknown function.

1. Introduction

Many engineering problems, such as adaptive control, pattern recognition, filtering, and identification, under the proper assumptions can be regarded as the parameter optimization problems. Consider a system with measurements $g(x, \alpha)$ on the performance function $M(\alpha) = E[g(x, \alpha)]$, where α is the parameter and x is the observation, the parameter optimization problem is to determine the optimal

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parameter α^* such that the performance function is extremized. The performance function $M(.)$ may also be a multi-modal function. Many efficient methods like steepest descent method, Newton's method, are available when the gradient ∇M is explicitly available. Usually, due to the lack of sufficient information concerning the structure of the function M or because of mathematical intractability, the function M to be optimized is not explicitly known and only noise-corrupted value of the function $M(\alpha)$ at any chosen point α can be observed.

Two important classes of algorithms are available for solving the above problem when only the noise-corrupted observations are available: stochastic approximation based algorithms [1] and learning automata based algorithms [2,3]. Stochastic approximation algorithms are iterative algorithms in which the gradient of the function M is approximated by a finite difference method, and using the function evaluations obtained at points, which are chosen close to each other [1]. The learning automata are adaptive decision making devices that operate in unknown random environments and progressively improve their performance via a learning process. Since learning automata theorists study the optimization under uncertainty, the learning automata are very useful for optimization of multi-modal functions when the function is unknown and only noise-corrupted evaluations are available. In these algorithms, a probability density function, which is defined over the parameter space, is used for selecting the next point. The reinforcement signal and the learning algorithm are used by the learning automata (LA) for updating the probability density function at each stage. We want that this probability density function converges to some probability density function where the optimal value of α is chosen with probability as close as to the unity. The distinguishing feature of the learning is that the probability distribution of $g(x, \alpha)$ is unknown. Methods based on stochastic approximation algorithms and learning automata represent two distinct approaches to the learning problems. Though both approaches involve iterative procedures, updating at every stage is done in the parameter space in the first method, which may result in a local optimum, and in the probability space in the second method. The learning automata methods have two distinct advantages over the stochastic approximation algorithms. The first advantage is that the action space need not be a metric space because as in the stochastic approximation algorithms in which the new value of the parameter is to be chosen close to the previous value. The second advantage is that the methods based on learning automata lead to global optimization, because at every stage any element of the action set can be chosen. Learning automata are divided into two main groups: finite action-set learning automata (FALA) and continuous action-set learning automata (CALA) based on whether the action set is finite or continuous [4]. FALA has finite number of actions and has been studied extensively. For an r-action FALA, the action probability distribution is represented by an r-dimensional probability vector that is updated by the learning algorithm. In many applications there is need to have large number of actions. The learning automata with too large number of actions converge too slowly. In such applications CALA, whose actions are chosen from real line, are very useful. For CALA, the action probability distribution is represented by a continuous function and this function is updated by learning algorithm at any stage. In the