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A direct hybrid finite element–wave based modelling technique for efficient analysis of poroelastic materials in steady-state acoustic problems

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Highlights

- A hybrid modelling technique for efficient analysis of Biot's poroelastic materials.
- Efficient analysis performance for addressing singularities in poroelastic domain.
- Domain with singularities and/or complex geometry discretised into finite elements.
- Geometrically moderate domain partitioned into convex wave based subdomains.
- Direct interface coupling of finite element and wave based poroelastic domains.

Abstract

This work proposes a hybrid modelling technique for efficient analysis of poroelastic materials, which are widely used for noise reduction in acoustic problems. By combining the finite element method and the wave based method in a direct manner, the proposed hybrid technique maximises the advantages and compensates the drawbacks of both numerical methods. The considered poroelastic domain described by Biot's theory is divided into two groups of domains according to their geometrical characteristics and boundary conditions. The group with complex geometries and/or boundary conditions leading to singularities is discretised into a large number of small finite elements. The other group consisting of large, geometrically moderate poroelastic domains is partitioned into wave based subdomains where the field variables are expanded with analytical poroelastic wave functions. Both groups modelled by the finite element method and the wave based method, respectively, are combined in a hybrid framework in this work to ensure their interacting dynamic behaviours. The properties of the hybrid model are investigated and are compared to existing modelling methods for some numerical examples. The proposed direct hybrid modelling technique provides stable

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predictions and exhibits fast convergence performances for the analysis of poroelastic materials, especially when singularities arise in the poroelastic domain.

Keywords: Hybrid modelling; Poroelastic materials; Wave based method; Finite element method; Biot's theory

1. Introduction

Sound has a powerful effect on humans and their environments as it determines to a large extent the general appraisal of products and spaces although these impressions may often be subjective. With increasing noise exposure in modern society, noise has been recognised as one of the most severe environmental hazards in our everyday life [1]. Recognition of this noise problem has produced many noise regulations [2–4], as well as growing consumer demands for quiet products. Although the direct treatment to block noise sources should be the first approach to solve noise problems, noise sources are often too diverse to control completely and/or simultaneously. Passive treatments that reduce noise transfers have shown to be effective in many cases and circumstances. Appropriate use of sound absorptive porous materials (mathematically described as poroelastic materials) in vibro-acoustic systems allows for noise from a variety of sources to be effectively dissipated such that it can no longer transfer to the target regions.

Because of the complicated physical phenomena involved and large degrees of freedom required for numerical implementation, poroelastic materials are often represented in merely phenomenological or mathematically simplified models for Computer Aided Engineering (CAE) applications. As poroelastic materials are key components for dissipation of acoustic energy in vibro-acoustic systems, however, there is a strong need for a development of efficient and accurate modelling technique to fully describe their dynamic behaviours to meet the regulations and demands of noise problems.

For the numerical modelling of poroelastic materials, conventional element based approaches, such as the finite element method (FEM) [5] and the boundary element method (BEM) [6], are most commonly used, similar to other applications in vibro-acoustic modelling. Since the element based approaches discretise the considered problem domains or their boundaries into a large number of small elements, geometrically complex domains can easily be represented. However, a larger number of smaller elements are required when considering higher frequencies as wavelengths shorten. Consequently, a huge increase of the degrees of freedom is inevitable to model large sized problems. Such high sensitivity to the system size (i.e., computation load) depending on the frequency of interest and the problem dimension makes the element based methods limited to relatively small sized problems and/or low-frequency ranges for practical CAE applications. This limitation is even more critical for poroelastic materials, which require a larger number of degrees of freedom in the numerical implementation due to their more complicated constitutional material behaviours and accompanying mathematical descriptions with their inherent strong frequency dependency and short wavelengths.

The wave based method (WBM) has been developed as an alternative deterministic approach to mitigate the difficulties in the element based models. Following the initial work by Desmet [7], wave based (WB) models provide efficient and accurate analysis of various steady-state vibro-acoustic problems, governed by generalised Helmholtz equations (e.g., see [8,9]). The WB approaches have quite advantageous properties as compared to the element based approaches. Based on an indirect Trefftz approach [10], the dynamic field variables in WB models are described using wave functions, which exactly satisfy the governing differential equations. As such, there is no approximation error inside the wave based domains in contrast to the interpolation and pollution errors [11,12] that arise for the element based approaches.

Instead of discretising problem domains into a number of small elements, WB models partition them into large convex subdomains, where the number and size are not dependent on the frequency of interest. Embedding a priori known information on the physics of the problem in the field variable description leads to a higher convergence rate for WB models. Therefore, WB systems are much smaller than those resulting from the element based approaches and consequently have a much lower computation load. On the other hand, WB models typically have a difficulty addressing geometrical complexity, because of the sufficient, but strong condition that the subdomains should be convex to guarantee convergence [7]. Thus, the WB models show to be most efficient for rather moderate geometries.