Mechanical Systems and Signal Processing 103 (2018) 216–235

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Delay-controlled primary and stochastic resonances of the SD oscillator with stiffness nonlinearities

Tao Yang, Qingjie Cao *

School of Astronautics, Harbin Institute of Technology, Harbin 150001, PR China

ARTICLE INFO

Article history:
Received 1 June 2017
Received in revised form 27 September 2017
Accepted 1 October 2017

Keywords:
SD oscillator
Stiffness nonlinearities
Time-delayed feedback
Primary resonance
Stochastic resonance

ABSTRACT

This work presents analytical studies of the stiffness nonlinearities SD (smooth and discontinuous) oscillator under displacement and velocity feedback control with a time delay. The SD oscillator can capture the qualitative characteristics of quasi-zero-stiffness and negative-stiffness. We focus mainly on the primary resonance of the quasi-zero-stiffness SD oscillator and the stochastic resonance (SR) of the negative-stiffness SD oscillator. Using the averaging method, we have been analyzed the amplitude response of the quasi-zero-stiffness SD oscillator. In this regard, the optimum time delay for changing the control intensity according to the optimization standard proposed can be obtained. For the optimum time delay, increasing the displacement feedback intensity is advantageous to suppress the vibrations in resonant regime where vibration isolation is needed, however, increasing the velocity feedback intensity is advantageous to strengthen the vibrations. Moreover, the effects of time-delayed feedback on the SR of the negative-stiffness SD oscillator are investigated under harmonic forcing and Gaussian white noise, based on the Langevin and Fokker-Planck approaches. The time-delayed feedback can enhance the SR phenomenon where vibrational energy harvesting is needed. This paper established the relationship between the parameters and vibration properties of a stiffness nonlinearities SD which provides the guidance for optimizing time-delayed control for vibration isolation and vibrational energy harvesting of the nonlinear systems.

1. Introduction

Vibration control is an everlasting research theme in mechanical field. In the last decade many researchers have investigated the vibration control technique of time-delayed dynamic vibration absorber. The underlaying idea is the introduction of an actuator controlled via state feedback with time delay. Time delay phenomenon appears in a finite signal transmission times, switching speeds, memory effects, etc. It can cause fundamental changes in dynamic characteristics to the system. Therefore, studying the effects of time delay not only plays a significant role in understanding the physical mechanism of control systems in the real world but also is of significant interest in the field of nonlinear science. So far, lots of efforts have been devoted to investigate the dynamics in dynamical systems with time delay, and lots of great results have been reported [1–9].

With the purpose of controlling vibration, a great deal of active control strategies are developed to improve the performance of vibrating systems [1–4]. It should be noted that, complex active control methods for vibration isolation or suppres-

* Corresponding author.
E-mail addresses: yangtscn@163.com (T. Yang), qingjiecao@hotmail.com (Q. Cao).

https://doi.org/10.1016/j.ymssp.2017.10.002
0888-3270/

sion are usually not preferable in practice because of its high energy cost and construction/installation expense. Because the effect of time delay control could be adjusted by changing the value of time delay without adding control gain, the time delay active feedback control attracts lots of attention for the benefit for vibration control performance. From the analysis of vibration suppression in literature about delayed feedback control, it is found that time delay could improve the vibration absorption. The concept of delayed-resonator vibration absorber is first proposed by Olgac et al. [5,6], and the servomotor could be used to introduce the delayed control signal for driving the vibration suppression performance of a vibration absorber. Hu et al. [7] studied the primary resonance and 1/3 subharmonic resonance of a forced Duffing oscillator with time-delayed state feedback control. It is demonstrated that the combination of positive displacement feedback and negative velocity feedback can effectively attenuate those resonances. Xu et al. [8,9] introduced the time-delayed feedback in high-static-low-dynamic vibration isolator to improve the system robustness and transmissibility performance.

The work in this paper is based on the research of a single-degree-of-freedom nonlinear SD (smooth and discontinuous) oscillator with a negative-stiffness mechanism, which was put forward by Cao et al. [10]. The SD oscillator was developed from a simple shallow arch model proposed by Thompson and Hunt [11], which is widely used in physics and engineering to illustrate the snap-through buckling phenomena. For the SD oscillator with stiffness nonlinearities, the system may have negative stiffness, zero stiffness as well as positive stiffness. On the one hand, to achieve the high-static-low-dynamic property in application, an isolator so-called quasi-zero-stiffness SD oscillator is proposed and studied [12–15]. The quasi-zero-stiffness SD oscillator could realize high static stiffness and ultra-low resonance frequency characteristics by appropriate geometrical design of nonlinearity. On the other hand, several interesting devices with the same or similar structures as the negative-stiffness SD oscillator have been designed, which present non-resonant large amplitude complex motions when excited with a sufficient intensity [16–20]. Consequently, they can optimize energy harvesting by using the material and geometrical nonlinearities. At present, the research on the vibration control of the geometrical nonlinear system is very insufficient. If the nonlinear systems are not designed or controlled properly, complex dynamic phenomena could be induced. Therefore, it is necessary to investigate the dynamical behaviours of a time-delayed system with geometric nonlinearity for gaining a deeper understanding of delay-induced nonlinear vibration in mechanical engineering, especially to provide the guidance for optimizing time-delayed control for vibration isolation and vibrational energy harvesting of the nonlinear systems.

As a matter of fact, the quasi-zero-stiffness SD oscillator can achieve vibration suppression over a broad frequency band based on its high-static-low-dynamic property, however strong nonlinear behaviors including multi-steady states and bifurcation phenomenon would induce by strong nonlinearity. Therefore, different control devices and methods are introduced to enhance the stability and improve vibration isolation effectiveness. The control mechanism is that reducing the resonance frequency to leave larger frequency band for isolation. The paper presents analytical study of the primary resonance of a harmonically forced quasi-zero-stiffness SD oscillator under state feedback control with a time delay. In addition, during the last couple of years, many researchers have also exploited purposefully introducing stiffness-type nonlinearities into the harvesters design. This has the influence of extending the coupling between the environmental excitation and the harvester to a wider range of frequencies [21–23]. The harvesting of energy from external sources, such as solar, thermal, wind, salinity gradients, and vibration, are primary candidates for development at a multiplicity of scales. Theoretical studies have clearly shown that the phenomenon of stochastic resonance (SR) can be used to generate large-amplitude vibrations in order to improve vibrational energy harvesting [16–19], but the development of practical methods for the precise control of SR is a further challenge to overcome. This paper presents a comprehensive analytical study of the delay-controlled SR in a negative-stiffness SD oscillator which can be designed for optimised mechanical energy harvesting.

SR is a well-known physical phenomenon that can be encountered in nonlinear systems where by a weak periodic modulating signal can act as a route to the amplification of the response of an oscillator driven by ambient vibration. The original work on SR by Benzi et al. [24,25] was in the context of modeling the switching of the Earths climate between ice ages and periods of relative warmth. After originally being proposed the conception of SR have been extended in recent decades, such as stochastic multi-resonance [26], double SR [27], superthreshold SR [28], quantum SR [29], entropic SR [30,31], spatiotemporal SR [32] and geometric SR [33]. Until now, the SR paradigm has made its presence felt in a wide range of applications, such as vibration energy harvesting [16–19], weak signal detection [34] and incipient fault diagnosis [35]. Besides, delay-induced SR have been observed in some classical models with the conclusion that delay can induce and control the SR [36–41]. This greatly enriched the applicability of the SR theory. Unfortunately, to the authors best knowledge, so far there is no result being reported on the problem of SR phenomenon in the negative-stiffness SD oscillator under state feedback control with a time delay. In addition, in many physical as well as stochastic systems, many studies indicate that the time delay and fluctuations exists in nature [42–45]. Noise is generally considered a harmful contribution in usual circumstances. Conversely, taking advantage of time delay and noise to keep and control active motion would be a powerful tool [46–49]. Actually, this would be possible by reaching the SR conditions in the SD oscillator. Up to this point, there remain many topics worth to be explored on SR in the negative-stiffness SD oscillator, especially for potential vibration energy harvesting.

In order to provide guidance of parameter design of the nonlinear SD oscillator with time-delayed control, the optimal value of time delay is explored for the system theoretically in this study. The main motivation is to combine the displacement and velocity feedback with a time delay to provide new insights into the complicated nonlinear dynamics of the SD oscillator with stiffness nonlinearities from the primary resonance and stochastic resonance, which include rich nonlinear phenomena and their occurrence mechanisms. This paper is organized as follows. Section 2 presents a SD oscillator model which can capture the qualitative characteristics of quasi-zero-stiffness and negative-stiffness. The nonlinear SD oscillator