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A combined averaging and frequency mixing approach for force identification in weakly nonlinear high-Q oscillators: Atomic force microscope

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ABSTRACT

We present a polynomial force reconstruction of the tip-sample interaction force in Atomic Force Microscopy. The method uses analytical expressions for the slow-time amplitude and phase evolution, obtained from time-averaging over the rapidly oscillating part of the cantilever dynamics. The slow-time behavior can be easily obtained in either the numerical simulations or the experiment in which a high-Q resonator is perturbed by a weak nonlinearity and a periodic driving force. A direct fit of the theoretical expressions to the simulated and experimental data gives the best-fit parameters for the force model. The method combines and complements previous works (Platz et al., 2013; Forchheimer et al., 2012 [2]) and it allows for computationally more efficient parameter mapping with AFM. Results for the simulated asymmetric piecewise linear force and VdW-DMT force models are compared with the reconstructed polynomial force and show a good agreement. It is also shown that the analytical amplitude and phase modulation equations fit well with the experimental data.

1. Introduction

In many physical and engineering experiments, the measurement indicates the presence of nonlinearity in the system. In order to have an idea about the structure or the physical parameters of the system producing the nonlinear response, system identification has to be carried out. System identification can be categorized in two classes: non-parametric and parametric. In the former class, no prior knowledge about the system structure is needed. The method searches for the best mathematical model in function space which is represented by a series of known functions, such that the model's behavior is similar to the original system subject to the same input. The series of known functions can be orthogonal or ordinary polynomials, or also special functions to cope with non-polynomial nonlinearity (e.g. hysteretic or piecewise nonlinearity). The second class, parametric identification, assumes that the mathematical structure or physical laws of the nonlinear system are known. The procedure then is to look for the suitable value in the parameter space, usually by the least squares method using the input/output information. Given the important role that system identification plays in helping to reliably predict the response of complex systems, numerous methods have been proposed to overcome the limitations of various applications. A general survey of papers and text books on nonlinear identification can be found in [3–6].

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An early non-parametric identification method proposed by Masri and Caughy [7], referred to as the restoring force surface (RFS) method, uses an orthogonal polynomial series expansion to fit the original restoring force which is considered as a function of displacement and velocity. Independently Crawley et al. [8] developed a variant of RFS for investigation of joints in space structures. Worden and Tomilson presented a simplification of RFS, and the RFS method was also used with ordinary polynomial series in [9,10]. Other approaches used the Harmonic Balance Method (HBM) to estimate the physical parameters [11–13]. The main idea is to expand the periodic response of the system in terms of truncated Fourier series with terms having frequencies that are integer multiple/sub-multiples of the excitation frequency. The unknown coefficients are obtained such that the resulting response from the model matches the one produced by original system. Mann and Khasawneh [14] presented a parametric identification method based on an energy-balance approach combined with cubic smoothing splines to avoid the occurrence of noise amplification due to numerical signal derivatives. In [11,13], the response was supposed to be steady state.

Boyd et al. developed a second order kernel transform measurement procedure using multi-tone harmonic probing. Chua and Liao [15] extended the procedure for third and higher order kernel transformation. Chatterjee and Vyas used the kernel procedure with multi-tone excitation to estimate parameters in Duffing oscillator [16]. Other authors employed Hilbert transform [17,18]. Some authors developed identification methods where the parameters are estimated from chaotic response [12,19–23]. Fitting the parameters to a chaotic time series has the advantage that the phase-space is better filled by a chaotic time series, than for example one periodic orbit [22]. When the data covers most of the phase space, the essential dynamical behavior is more likely to be captured. Other authors have considered chaotic excitations [24,25]. Doughty et al. [13] compared three techniques using steady state data to identify an externally excited cantilever beam. One of the techniques, which is related to the method presented in this paper, was based on the multiple time scales analysis, where the data were fitted to the amplitude and phase modulation equations, and resonance was exploited as in [26].

There exist some ways of improving system identification, for example exciting the system such that the output data are strongly dependent on the nonlinearity [27,28]. Multi-harmonic excitation was adopted as a way to enhance response generated by the nonlinearities present in the physical system [21,29]. Applying multi-tone excitation leads to intermodulation in the response components within the measurement bandwidth, thereby improving the parameters fitting process in comparison with single-tone excitation [2].

In the atomic force microscopy the interaction forces between the tip and the sample are extracted or reconstructed from the measured frequency, amplitude or phase shift as the oscillating probe is brought closer to the sample surface. The interaction forces in the AFM experiment allow knowledge about the mechanical, physical and chemical properties of the sample, e.g. chemical composition, elasticity and friction. Therefore knowing these interaction forces helps to characterize the studied sample. Platz et al. [1] presented a physical interpretation of the tip motion by considering two force quadratures F_I and F_Q, where the former is in-phase with the cantilever motion while the latter is out-of-phase. The same authors used a polynomial force approximations, extracting the polynomial coefficients from the two force quadratures [30]. Payam et al. [31] used tools from fractional calculus with the assumption that the amplitude and phase shift are slowly varying functions of the tip-surface separation to reconstruct the interaction forces.

In this work we reconstruct nonlinear forces in a high-Q oscillator by combining the amplitude and phase modulation equations obtained from the method of averaging and the slowly varying envelope function that is obtained directly from the measured narrow-band frequency comb. The paper is organized as follows: We begin by describing the method and presenting the averaging procedure and the slowly varying envelope analysis. In Section 3 we consider the Atomic Force Microscope example, first using an asymmetric piecewise linear force and VdW-DMT force models to compare with a reconstructed polynomial force. Then we consider data obtained from an AFM experiment and we reconstruct the tip-sample force. Our conclusions are briefly stated in Section 4.

2. Description of the method

We consider an oscillator with high quality factor Q that is subjected to two drive forces and perturbed by nonlinear forces. The equation of motion of such oscillator takes the following form:

$$m\frac{d^2y}{d\tau^2}+\frac{m\Omega_0}{Q}\frac{dy}{d\tau}+ky=\bar{F}_1\cos\Omega_1 t+\bar{F}_2\cos\Omega_2 t+\bar{F}_{\mathrm{NL}}\left(\bar{\mu}_i,y,\frac{dy}{d\tau}\right), \tag{1}$$

where y represents the displacement of the oscillator, m the mass, Q the quality factor, k the stiffness, and Ω_0 the natural frequency. $\bar{F}_{1,2}$ and $\Omega_{1,2}$ are the amplitude and the frequency of the forcing drivers, respectively. $\bar{F}_{\mathrm{NL}}$ is a nonlinear interaction force between the oscillator and the surrounding, described by the parameters $\bar{\mu}_i$. For analytical convenience we nondimensionalize Eq. (1)

$$\frac{d^2x}{dt^2}+\delta\frac{dx}{dt}+w_0^2x=F_1\cos t+F_2\cos w_2 t+F_{\mathrm{NL}}(\mu_i,x,\dot{x}), \tag{2}$$