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Exploiting input sparsity for joint state/input moving horizon estimation

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This paper proposes a novel time domain approach for joint state/input estimation of mechanical systems. The novelty consists of exploiting compressive sensing (CS) principles in a moving horizon estimator (MHE), allowing the observation of a large number of input locations given a small set of measurements. Existing techniques are characterized by intrinsic limitations when estimating multiple input locations, due to an observability decrease. Moreover, CS does not require an input to be characterized by a slow dynamics, which is a requirement of other state of the art techniques for input modeling. In the new approach, called compressive sensing–moving horizon estimator (CS-MHE), the capability of the MHE of minimizing the noise while correlating a model with measurements is enriched with an ℓ_1-norm optimization in order to promote a sparse solution for the input estimation. A numerical example shows that the CS-MHE allows for an unknown input estimation in terms of magnitude, time and location, exploiting the assumption that the input is sparse in time and space. Finally, an experimental setup is presented as validation case.

1. Introduction

State estimation is a well established engineering approach aiming at recovering a complete representation of the internal condition of the system under investigation at a given time instant, and allows a system to be controlled [1]. This is achieved by correlating information coming from a model with a set of measurements. As such, it is becoming crucial in many engineering areas such as control, structural health monitoring and virtual sensing. It is well known that the Kalman filter (KF) [2] provides the optimal state estimation in case of linear systems with Gaussian, zero-mean, uncorrelated process noise [1]. However, many real world systems do not satisfy these hypothesis, and other estimation methodologies have been developed, such as the extended Kalman filter (EKF) [1] and the moving horizon estimator (MHE) [3]. The KF and its nonlinear derivations are recursive single step approaches, whereas the MHE exploits a finite length time window sliding over time. Moreover, the MHE is suited for nonlinear systems, can include constraints and has been shown to provide the correct estimation for problems with multiple optima where the EKF tends to fail [4], at the price of a higher computational cost.

All the over mentioned techniques can also combine the estimation of states and inputs. In such framework, the inputs become part of the unknowns and are referred to as augmented states [1,5]. A joint state/input estimator is beneficial

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whenever the inputs are not easy to be measured or if they have a strong influence on the estimation accuracy [1,6–8]. In fact, a joint estimator can capture the cross-coupling among all estimates by means of a single covariance matrix. However, this comes at the price of a higher computational cost and possible observability issues, which may degenerate to failure of estimation when dealing with many estimates [1]. Furthermore, additional equations are needed to model these new states. A random walk model is often employed to represent an unknown input. It is a generic approach that can be applied to different input types, but it is not suited if the number of augmented states exceeds a threshold governed by observability or if the new states are characterized by a fast dynamics [1,6–8,5,9–12].

Beside estimation problems, input models are key elements of force identification problems. Refs. [13–16] focus on inverse methods for force identification based on the ℓ_1-norm regularization. In particular, reference [15] describes a fast iterative shrinkage/thresholding (IST) algorithm which leads to an accurate force reconstruction for impulses and harmonic loads. The same approach is employed in [14], where different types of shape functions are compared in order to find the best force impact representation. Furthermore, the problem of impact identification and location is solved in [13] by a two-step IST algorithm, allowing the characterization of one or two force impulses within a set of nine candidate locations. Finally, reference [16] describes the sparse deconvolution method for the reconstruction of impact forces in case of large scale ill-posed inverse problems.

The ℓ_1-norm optimization constitutes the basis of a technique called compressive sensing (CS), which is gaining attention in the fields of structural health monitoring and fault detection in order to limit the number of required sensors [17–19] and the amount of data to be transmitted for processing [20]. CS allows to acquire and recover undersampled signals, and it is based on a concept referred to as signal sparsity [21–23]. CS is currently being investigated as a powerful instrument in the framework of estimation problems based on the KF. The first example of CS as a tool to improve the KF can be found in [24], where the fact that the sparsity pattern of a signal changes slowly over time is exploited within a KF with a limited amount of measurements. This idea has been further developed in [25,26], and two other sparsity conditions have been introduced in [27] (*i.e.*, sparsity in the state and sparsity in the innovations) in order to improve the KF performances in terms of estimation error or convergence time.

Ref. [28] applies CS for the detection of a single force impact entering a mechanical system at an unknown location, such that the signal is known to be sparse in time and space, and CS allows for an accurate input estimation. Input sparsity in space has also been exploited in [29], where a frequency domain approach is proposed to identify unknown dynamic forces on a structure.

To summarize the state of the art, a joint state/input estimator is a powerful tool if the knowledge of an input is crucial to the estimation accuracy and cannot be obtained from direct measurements. However, such approach is not suitable if the number of inputs exceeds an observability constraint or a random walk is chosen to represent a high dynamics input. Recent research shows that CS can lead to an accurate force reconstruction within a static time horizon, independently from the input dynamics, provided that a set of basis functions is available to guarantee input sparsity. CS was also employed to improve a KF, proving that the sparsity within a single time step can be propagated to the next step in an iterative fashion.

This paper describes a novel joint state/input estimation approach which limits the observability issues related to the estimation of multiple variables and can be employed for the estimation of inputs characterized by a fast dynamics, thus confining the drawbacks of state augmentation with respect to observability and overcoming the limitations of the random walk model. This new methodology will be referred to as the compressive sensing–moving horizon estimator (CS-MHE). The CS-MHE is a joint state/input estimator in which an unknown input is modeled as a sparse signal. In particular, the CS-MHE exploits input sparsity in time and space. This is achieved by projecting an input onto a set of basis functions of which only a few are active. Thus, inputs distributed in time and/or space can be estimated provided that an appropriate set of basis functions is available [30].

The paper is structured as follows: first, Section 2 gives an overview of the MHE and on CS. Next, Section 3 illustrates the proposed CS-MHE methodology. The CS-MHE is tested first numerically in Section 4 and then experimentally in Section 5 for a linear time-invariant (LTI) mechanical system loaded with an impact force, proving that the CS-MHE allows to detect an unknown input on a mechanical system (*e.g.*, a force) in terms of position, magnitude and time. Finally, Section 6 summarizes the conclusions.

2. An overview of MHE and CS

This section gives an overview of the two milestones on which the CS-MHE is based, *i.e.*, the moving horizon estimator (Section 2.1) and compressive sensing (Section 2.2).

2.1. Moving horizon estimator

Refs. [4,3,31] describe the classical MHE for state estimation problems, which is summarized by Eqs. (1)–(5). An estimation window of length N is defined between the discrete time steps $k = T - N + 1$ and $k = T$, as shown in Fig. 1.