Mechanical Systems and Signal Processing 101 (2018) 320–337

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Iterative algorithms for the input and state recovery from the approximate inverse of strictly proper multivariable systems

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ARTICLE INFO

Article history:
Received 23 February 2017
Received in revised form 19 June 2017
Accepted 14 August 2017

Keywords:
Approximate inverse
Dummy *D* matrix
Iterative algorithm
Stability
Strictly proper

ABSTRACT

This paper proposes new iterative algorithms for the unknown input and state recovery from the system outputs using an approximate inverse of the strictly proper linear time-invariant (LTI) multivariable system. One of the unique advantages from previous system inverse algorithms is that the output differentiation is not required. The approximate system inverse is stable due to the systematic optimal design of a dummy feedthrough *D* matrix in the state-space model via the feedback stabilization. The optimal design procedure avoids trial and error to identify such a *D* matrix which saves tremendous amount of efforts. From the derived and proved convergence criteria, such an optimal *D* matrix also guarantees the convergence of algorithms. Illustrative examples show significant improvement of the reference input signal tracking by the algorithms and optimal *D* design over non-iterative counterparts on controllable or stabilizable LTI systems, respectively. Case studies of two Boeing-767 aircraft aerodynamic models further demonstrate the capability of the proposed methods.

1. Introduction

The inverse of a linear time-invariant (LTI) multi-in/multi-out (MIMO) system, which can be used to recover unknown inputs from the system outputs, was extensively studied in past decades due to its importance and broadness in industrial applications, such as the fault detection and isolation (FDI) [1–3], fault-tolerant control (FTC) [4], filtering and coding theory. Brockett and Mesarović first brought in the concept of reproducibility of a system in their seminal paper [5] in 1965, which is later redefined as an inverse problem. Building on their work, invertibility conditions and system inverse algorithms were developed by Sain and Massey [6,7], Dorato [8], Silverman [9], Porter [10], Moylan [11], Hirschorn [12], to name a few. Specifically, Sain and Massey brought in the 'inherent integration' to check the system invertibility; Silverman developed the 'structure algorithm' which was suitable for both time-invariant and time-varying systems, and Hirschorn [12] extended linear inverse results to real analytic nonlinear systems.

Common features of the above algorithms are that (i) Output differentiation (dy/dt) was required which might suffer the loss of accuracy when outputs are noisy; (ii) A feed-through *D* matrix was presented in the state-space system described by the matrix fourtuple (A, B, C, D), i.e., at least one entry in *D* is nonzero although a singular or nonsquare *D* is allowed. Unfortunately, due to the time delay, a physical process usually has strictly proper transfer functions which inherently lack such a *D* matrix in the system description. On the other hand, Skogestad and Postlethwaite [13] suggested that an approximated

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http://dx.doi.org/10.1016/j.ymssp.2017.08.026
0888-3270/

inverse of such a strictly proper system could be obtained by creating an approximate system with an additional 'dummy' feed-through D matrix with small entries added to the original state-space system. Such a small dummy D matrix, however, should be chosen not to result in any right-half plane (RHP) zeros causing the instability of the inverse system. It seems like an easy fix at the first glance, nevertheless, a deeper investigation reveals that such a D may not always exist and tremendous trial and error could be involved in the search of it, especially when the size of the system and thus the size of D become large. Moreover, even such a D is found, the reference tracking performance of the recovered inputs from the approximate system have not been addressed. Therefore, there still lacks a systematic and reliable way to identify the inverse of a strictly proper system with an accurate estimation of the system inputs.

In view of these problems, we introduce two correlated iterative inverse algorithms for the purpose of system inverse construction and input recovery from the strictly proper LTI MIMO system. Specifically, the "every-time" algorithm, in which iterations are performed at each time point, is capable of recovering various bounded and continuous input signals, while the "point-time" algorithm is based on a certain time point at steady-state which can rapidly recover unknown step inputs with some other benefits. Both algorithms are computationally efficient where no output differentiations are needed. A systematic procedure avoiding the time-consuming trial and error is also proposed to identify the inverse of strictly proper systems known as the optimal design of dummy D matrix. The remainder of this paper is arranged as follows. Section 2 covers the approximate inverse system construction and algorithm development. Section 3 defines two types of convergence and discusses the convergence criteria for the algorithms which would be used as the constraints in the optimal dummy D design. Section 4 introduces the optimal dummy D design for controllable and stabilizable (stable but uncontrollable) systems, respectively. Illustrative examples are then provided in Section 5 to demonstrate the efficacy and superiority of the proposed algorithms and dummy D design over the aforementioned trial and error small D method in both systems. Then, two aircraft aerodynamic state-space models are used in Section 6 to further demonstrate its capability for with various testing signals. Finally, the concluding remarks are summarized in Section 7.

2. Algorithm development

2.1. System definitions

Consider the strictly proper LTI system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(t) \in \mathbb{R}^n,\ u(t) \in \mathbb{R}^p \tag{1}$$

$$y(t) = Cx(t), \quad y(t) \in \mathbb{R}^p \tag{2}$$

where A, B and C are matrices with appropriate dimensions. Without the loss of generality, initial states are assumed to be zero first, i.e., $x(0) = 0$ and nonzero situation will be discussed in Section 3.2. The corresponding transfer function matrix in the frequency domain is given by $G_s(s)$, which is strictly proper. From Eqs. (1) and (2), the Laplace transform of the output $Y(s)$ is

$$Y(s) = G_s(s)U(s) = C(sI - A)^{-1}BU(s) \tag{3}$$

Although the inverse of $G_s(s)$ is improper and thus cannot be realized by state-space model [14], a nonsingular dummy D can be created to make the new transfer function matrix $G(s)$ proper and realizable which generates the same output $Y(s)$ shown by (4). The corresponding state-space model is represented by (5) and (6).

$$Y(s) = G(s)\hat{U}(s) = (C(sI - A)^{-1}B + D)\hat{U}(s) \tag{4}$$

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\hat{u}(t), \quad (\hat{x}(0) = 0), \hat{x}(t) \in \mathbb{R}^n, \hat{u}(t) \in \mathbb{R}^p \tag{5}$$

$$y(t) = C\hat{x}(t) + D\hat{u}(t), \quad y(t) \in \mathbb{R}^p \tag{6}$$

Pre-multiplying D^{-1} on (6) and substitute $\hat{u}(t)$ into (5) gives the inverse system shown by (7) and (8). The corresponding transfer function matrix $(G(s)^{-1})$ is represented by (9), in which $\hat{U}(s)$ will be called the estimated input in the rest of the paper. It is generally assumed that $U(s)$ is bounded in the frequency range of study.

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}y(t), \tag{7}$$

$$\hat{u}(t) = \hat{C}\hat{x}(t) + \hat{D}y(t) \tag{8}$$

$$\hat{U}(s) = G(s)^{-1}Y(s) = (\hat{C}(sI - \hat{A})^{-1}\hat{B} + \hat{D})Y(s) \tag{9}$$

where $\hat{A} = A - BD^{-1}C, \hat{B} = BD^{-1}, \hat{C} = -D^{-1}C, \hat{D} = D^{-1}$.