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L_p-norm minimization for stochastic process power spectrum estimation subject to incomplete data

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ABSTRACT

A general L_p norm ($0 < p \leq 1$) minimization approach is proposed for estimating stochastic process power spectra subject to realizations with incomplete/missing data. Specifically, relying on the assumption that the recorded incomplete data exhibit a significant degree of sparsity in a given domain, employing appropriate Fourier and wavelet bases, and focusing on the L_1 and $L_{1/2}$ norms, it is shown that the approach can satisfactorily estimate the spectral content of the underlying process. Further, the accuracy of the approach is significantly enhanced by utilizing an adaptive basis re-weighting scheme. Finally, the effect of the chosen norm on the power spectrum estimation error is investigated, and it is shown that the $L_{1/2}$ norm provides almost always a sparser solution than the L_1 norm. Numerical examples consider several stationary, non-stationary, and multi-dimensional processes for demonstrating the accuracy and robustness of the approach, even in cases of up to 80% missing data.

1. Introduction

Reconstruction of discrete time/space signals that suffer from missing data has long been a topic of interest across a range of fields. Whilst the most effective way to address such problems is to sample signals more reliably, under controlled conditions, this is not always possible. "Missing data" in general, refers to situations in which undesirable gaps occur in data sets. For example, in practice, such problems may be caused by sensor failures or sampling/threshold limitations on the equipment, acquisition or usage restrictions on sensing or on the data itself, and even from data corruption. Re-sampling missing data can be difficult in many cases, and often impossible when working with non-stationary stochastic processes. For this reason, there are numerous approaches to addressing these problems by predicting missing datum values based on the available data. These include zero-padding of missing data [1], least-squares spectral analysis [2–4], iterative spectral de-noising [5–7], interpolative as well as autoregressive methods [8]. Clearly, in most cases the choice of the approach is problem-dependent, and typically depends on a priori known information such as the arrangement and amount of missing data. This paper focuses on a class of missing data problems for which the property of "sparsity" is exploited to reconstruct records. A sparse discrete-time signal can be characterized by a relatively small number of coefficients with respect to its sample length. This sparsity may be apparent in the sampling domain, for which the majority of the data is zero except

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for a handful of spikes, or sparsity can occur in some other basis or frame, such as the frequency domain. Signal reconstruction methods that take advantage of sparsity have received increased interest with the advent of Compressive Sensing (CS) [9,10], a signal processing technique in which data are purposely under-sampled.

Regarding applications in structural engineering/dynamics, so far CS has been mostly applied in situations where some saving in data capture time or data size is useful. For example, sensors (especially wireless ones) that capture data for real-time structural health monitoring can be designed to capture only a fraction of the data, reducing manufacturing cost. By utilizing CS with an appropriate compression basis (in which the signal has a sparse representation), data series with far higher resolution than those originally captured could be reconstructed. Not only would the sensors not need to capture as much data, but also the stored data would have a small file size, negating the requirement for compression processing at the sensor. In this regard, some preliminary recent results exist in the literature for structural system parameters identification, damage detection and health monitoring [11–21]. However, most of the aforementioned applications are restricted in the sense that they are focused on the problem of compressing efficiently the acquired signal (assumed to be complete) for circumventing the computational burden of compressing it locally at the sensor. Nevertheless, applying CS theory to the problem of missing data differs primarily in one respect; that is, missing data are not necessarily intentional. Unfortunately, this removes control over one important step of CS: the arrangement of the sampling matrix. CS relies on the choice of an appropriate sampling matrix. For instance, uniform random Fourier matrices obey the CS requirements for sparse reconstruction with high probability [9,10]. Unfortunately, the missing data may not be uniformly distributed over the record; thus, regular or large gaps of missing data can lead to lower orthogonality between random columns of the sampling matrix. Further, even the papers that address the case of data losses such as in [12], focus primarily on deterministic signal reconstruction (e.g. in the time domain). Nevertheless, there are cases (e.g. system reliability assessment applications) where the main objective may not be signal reconstruction (in the time/space domains), but rather characterization and quantification of the underlying stochastic process/field statistics (i.e. Power Spectrum estimation).

Recently, the authors utilized sparse signal reconstruction methods to develop stochastic process power spectrum estimation techniques subject to signals with missing data [22]. The concept of the power spectrum has been indispensable for characterizing stochastic processes that exhibit frequency-dependent properties (e.g., [23–25]). Nevertheless, to estimate the power spectrum of a stochastic process, recorded realizations are often required, which may suffer from previously mentioned missing data problems. Note that power spectrum estimation methods that rely on the Discrete Fourier Transform (DFT) or on wavelet transforms for the non-stationary case, require full, uniformly sampled data sequences; hence the need for reconstruction. In this regard, many processes for which a power spectral model is of interest exhibit relative sparsity in the frequency domain, and thus, sparse reconstruction methods can be ideal. In [26], a CS based approach was developed for power spectrum estimation, in which multiple records were utilized to iteratively update a harmonic basis matrix, demonstrating significantly improved results over alternative methods, and has been applied in the context of structural response and reliability analysis [27]. Further, it is noted that for both stationary and non-stationary processes for which only single records are available, windowing and down-sampling may be applied to emulate multiple process records.

In the above contributions [26,27], L_1 norm minimization was utilized for signal reconstruction, which is commonly applied within a CS framework. In this paper, the power spectrum is estimated by utilizing an alternative $L_{1/2}$ norm minimization procedure. This is more likely to lead to sparser signal reconstruction, with enhanced accuracy. Set within the aforementioned iterative scheme [26], and assuming that multiple process records are available for analysis, $L_{1/2}$ norm minimization solutions are presented alongside L_1, demonstrating the effect of enhanced sparsity upon the "mean" spectrum. It is important to note, however, that there exist several alternative reconstruction schemes in the literature that make use of the L_p $(0 < p \le 1)$ norm [28], or approach the problem from a probabilistic perspective such as Bayesian compressive sensing (BCS) [29,30]. The latter is able to provide a measure of noise present in the record and estimate the error in the reconstruction. In this regard, BCS may present an ideal tool for use in conjunction with the iterative basis re-weighting utilized herein, where the relationship between error in the individual reconstruction vs the error in the ensemble estimated spectrum is of prime interest.

The following section comprises a brief background to identification of sparse solutions via L_p norm $(0 < p \le 1)$ minimization schemes. Further, it provides an overview of the L_1 norm re-weighting procedure that utilizes multiple stochastic process records for power spectrum estimation described in detail in [26]. The re-weighting procedure is then utilized alongside $L_{1/2}$ norm minimization, further promoting sparsity. Both methods are then compared for varying numbers of available process records for stationary, non-stationary and multi-dimensional cases.

2. Sparse solutions via L_p norm minimization

The condition of sparsity requires that a signal can be defined in some known basis with far fewer coefficients than the number determined by the Shannon-Nyquist rate [31]. As an example, a discrete time signal x in one dimension can be viewed as an $N \times 1$ column vector. Given an orthogonal $N \times N$ basis matrix A, in which the columns A_i are the basis functions, x can be represented in terms of this basis via a set of $N \times 1$ coefficients y, i.e.,

$$x = \sum_{i=1}^{N} A_i y_i, \tag{1}$$