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A fractional-order accumulative regularization filter for force reconstruction

Jiang Wensong [a,*], Wang Zhongyu [a], Lv Jing [b]

[a] School of Instrumentation Science and Opto-electronics Engineering, Beihang University, Beijing 100191, China
[b] China National Accreditation Service for Conformity Assessment, Beijing 100062, China

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ABSTRACT

The ill-posed inverse problem of the force reconstruction comes from the influence of noise to measured responses and results in an inaccurate or non-unique solution. To overcome this ill-posedness, in this paper, the transfer function of the reconstruction model is redefined by a Fractional order Accumulative Regularization Filter (FARF). First, the measured responses with noise are refined by a fractional-order accumulation filter based on a dynamic data refresh strategy. Second, a transfer function, generated by the filtering results of the measured responses, is manipulated by an iterative Tikhonov regularization with a serious of iterative Landweber filter factors. Third, the regularization parameter is optimized by the Generalized Cross-Validation (GCV) to improve the ill-posedness of the force reconstruction model. A Dynamic Force Measurement System (DFMS) for the force reconstruction is designed to illustrate the application advantages of our suggested FARF method. The experimental result shows that the FARF method with $r = 0.1$ and $\alpha = 20$, has a PRE of 0.36% and an RE of 2.45%, is superior to other cases of the FARF method and the traditional regularization methods when it comes to the dynamic force reconstruction.

1. Introduction

Measuring the dynamic force acting on a mechanical structure is a critical aspect of the industrial applications such as the structural health monitoring, the mechanical property detection, the force transducer calibration, the force reproduction, and some other load testing cases [1–4]. However, forces cannot always be sampled at any conditions or on any locations by a direct measuring instrument in actual applications [5].

For this reason, the force reconstruction is increasingly becoming a vital supplementary approach to determine the dynamic force from measured structural responses, such as acceleration, velocity, displacement, and strain [6]. In details, the force reconstruction happens when the tested body cannot be accessed by a force transducer [7], the test locations are limited versus a large size surface [8], and other practical applications that direct measuring methods are unavailable [9]. In the field of solid mechanics, the force reconstruction is typically referred to as an inverse problem that the structural responses are measurable while the forces acting on the system are unknown.

So far, there are three methods have been developed for the force reconstruction, such as the direct method, the regularization method, and the probabilistic/statistical method.

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[*] Corresponding author.

E-mail addresses: jwensong@163.com (J. Wensong), mewan@buaa.edu.cn (W. Zhongyu), lvj@cnas.org.cn (L. Jing).

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The direct method can build the inverse problem by direct using the physical or the mathematical model of the object in the time domain or the frequency domain. Jacquelin and Hamelin applied this method to a split Hopkinson pressure bar by utilizing strain measurements [10]. HE and his researchers reconstructed the impact forces among the debris flow by direct analyzing the composition of its particle materials based on a hydrodynamic and contact mechanical theory [11]. Parzianello and Francesconi reconstructed the local forces of a plate subjected to a hypervelocity impact by using the finite element analysis [12]. Unfortunately, since these cases of inverse problem are manipulated without any filter or any additional constraint, the reconstructed force has varying levels of accuracy. That is because the force reconstruction is often an ill-posed problem, where a small noise in measured responses can generate a significant error in the desired solution.

To overcome this ill-posedness, the regularization method has been developed for the force reconstruction. It consists of a Tikhonov regularization, a modal truncation approach, and an optimization approach. For example, Jiang and Hu proposed the model selection techniques to reconstruct the dynamic load acting on a Euler beam and a thin plate [13,14], Djamaa and his colleagues developed a frequency range truncation method to reconstruct the force acting on a thin cylindrical shell [15]. In addition to the modal truncation, the optimization approach can solve the optimal estimation of the system input by minimizing the prediction error of a given problem. Aucejo and DeSmet proposed a multiplicative regularization to optimize the regularized solution iteratively [16]. The conjugate gradient algorithm is another type of optimization approach which was detailed by Yen and Wu [17,18] and was further applied by Qiao and his fellow scholars [19,20] to solve a modified sparse deconvolution model. The Levenberg-Marquardt iterative regularization method is also an optimization method to solve inverse problems, and were presented by Gasparo et al. [21] and Jiang et al. [22]. Although these regularization methods are robust and their applications are generally successful, they have their drawbacks. For example, it's hard to determinate the additional parameters of the regularization in an accurate way, and a neglect of noise could result in a significant error in the estimate of the applied force.

Considering the uncertainties of measurement noise into the analysis, the probabilistic/statistical method has been applied to the solution of the inverse problem in a more holistic perspective. As a prevalent algorithm for the inverse problem of the force reconstruction, the Kalman filter has been applied in an augmented form by Lourens and his team members [23], in incorporation with a recursive least squares algorithm by Ma and his colleagues [24]. Other methods such as asymptotic approximation method proposed by Sanchez and Benaroya [25], regularized Wiener filter method by Gunawana [26], adaptive neuro-fuzzy approach by Dalibor et al. [7], and pseudo-inverse method by Khoo and his workmates [27] are increasingly becoming prevalent in the application of force reconstruction. In recent years, the Grey Model is also increasingly becoming a novel statistical approach to signal preprocessing by Wu et al. [28] and ill-posed problem manipulation by Ma and his team members [29]. This approach has been applied in a fractional-order form by Atherton [30], in a non-equidistant fractional order form by Jiang [31], and other noise processing cases [32,33].

Since the ill-posed system is sensitive to noise, the force reconstruction methods that account for noise may lead to the accurate and self-contained results. Thus, algorithms that better control the system noise with a more rigorous analysis would be exceedingly beneficial to the force reconstruction. The objective of this paper is to develop a dynamic force reconstruction method that is stable to noise by a hybrid of statistical and regularization method.

This paper is focused on solving the ill-posed inverse problem of the force reconstruction on a DFMS by using the fractional order accumulative regularization filter method and organized as follows. In Section 2, the inverse problem of the force reconstruction on a DFMS is developed. In Section 3, the ill-posed force reconstruction problem is regularized by the FARF method, which involves a filtering process and a regularization process. In Section 4, the numerical simulations are created to assess the FARF method. In Section 5, the exciting forces acting on the DFMS are reconstructed by the FARF method. Both the effectiveness and the accuracy of the FARF method are comparatively discussed with the traditional force reconstruction method. Finally, several conclusions and research expectations about our work are given in Section 6.

2. Overview of the force reconstruction

2.1. Research object: force reconstruction of the DFMS

As an important part of the National Quality Infrastructure (NQI), the reproducibility/traceable measurement of the dynamic force is realized by the DFMS [34,35]. The DFMS consists of an acceleration sensor, a Free Falling Hammer (FFH), and a force transducer, as Fig. 1 shows. In general, an exciting force $f(t)$ will be generated from the source point when the FFH impacts and stops on one end of the force transducer. Meanwhile, the acceleration sensor on the target point will output a measured response $s(t)$.

Since the complex structure of the FFH, however, the exciting force on the source point is unlikely the same as the measured force on the target point. This difference will accumulate into a force reproducibility error when the traditional measuring method is used. To correct this reproducibility error, the exciting force $f(t)$ on the source point needs to be reconstructed based on the measured response $s(t)$ on the target point.

In ideal situations, the measured response to a single force can be solved by the convolution integral between the Green transfer function and the exciting force, it is defined as

$$s(t) = \int_0^t f(\tau)g(t-\tau)d\tau \qquad (1)$$