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Three new models for evaluation of standard involute spur gear mesh stiffness

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ABSTRACT

Time-varying mesh stiffness is one of the main internal excitation sources of gear dynamics. Accurate evaluation of gear mesh stiffness is crucial for gear dynamic analysis. This study is devoted to developing new models for spur gear mesh stiffness evaluation. Three models are proposed. The proposed model 1 can give very accurate mesh stiffness result but the gear bore surface must be assumed to be rigid. Enlighted by the proposed model 1, our research discovers that the angular deflection pattern of the gear bore surface of a pair of meshing gears under a constant torque basically follows a cosine curve. Based on this finding, two other models are proposed. The proposed model 2 evaluates gear mesh stiffness by using angular deflections at different circumferential angles of an end surface circle of the gear bore. The proposed model 3 requires using only the angular deflection at an arbitrary circumferential angle of an end surface circle of the gear bore but this model can only be used for a gear with the same tooth profile among all teeth. The proposed models are accurate in gear mesh stiffness evaluation and easy to use. Finite element analysis is used to validate the accuracy of the proposed models.

1. Introduction

Gearboxes are the most commonly used transmission system, which can provide speed and torque from a rotating power source to another device [1]. To meet various application purposes, many types of gears are designed, for example, spur gear, helical gear, face gear, bevel gear, worm gear, hypoid gear and screw gear. Spur gears are the simplest but the most widely used type of gears. For a pair of spur gears without tooth profile modifications, one pair and two pairs of tooth contacts take place alternatively. In addition, the tooth contact position changes with time. Therefore, the gear mesh process is very complicated even for involute spur gear pairs. Fig. 1 illustrates these two gear meshing scenarios. Fig. 1(a) shows the situation of one pair of teeth in mesh while Fig. 1(b) presents the scenario of two pairs of teeth in mesh. The points $\boldsymbol{P}$, $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ in Fig. 1 are the mesh points. There are no tooth contacts in other positions of Fig. 1 even though some points appear to be in touch. For involute spur gear pairs, the mesh points move along the action line with the rotation of gears. The action line is the common tangent to the base circles of the pinon and the gear, and the common normal to the tooth profiles [2].

Gear dynamics can be used to improve gear quality in the design process and reduce the failure rate of gears during operation [3]. A main internal excitation source of gear dynamics is time-varying gear mesh stiffness [4,5]. According to the lit-

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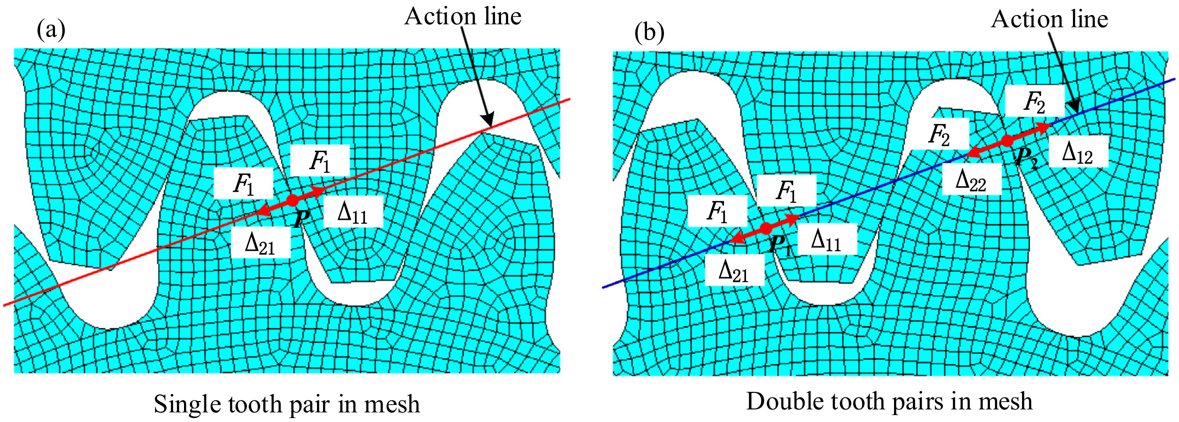

Fig. 1. Mesh contact scenarios of a pair of external spur gears.

erature, there are two types of stiffness for a pair of gears, rectilinear mesh stiffness and torsional mesh stiffness [6]. Rectilinear mesh stiffness is an equivalent mesh stiffness of a pair of gears along the action line [7]. Torsional mesh stiffness is defined as the ratio between a torque applied on a gear (the mating gear's body is fixed) and the corresponding angular displacement of the gear body [6]. Both rectilinear mesh stiffness and torsional mesh stiffness are time-varying due to the change of tooth contact position and the number of tooth pairs in simultaneous mesh [8]. These two types of mesh stiffness are related to each other [6]. In gearbox dynamic modeling, the stiffness between two meshing teeth is the rectilinear mesh stiffness [6,9]. Some researchers obtain the rectilinear mesh stiffness from the torsional mesh stiffness [6]. The relationship between the rectilinear mesh stiffness and the torsional mesh stiffness will be analyzed in detail in this study.

Accurate evaluation of gear mesh stiffness is essential for gear dynamics and vibration analysis [9,10]. Rectilinear mesh stiffness can be evaluated using the potential energy method [7,11–15], the finite element method [5,16,17,18], or the experimental methods [19–21]. Torsional mesh stiffness is generally evaluated using the finite element method [6,22–25].

In the potential energy method, the gear tooth is modeled as a non-uniform cantilever beam and the Timoshenko beam theory [26] is used. The total energy stored in a pair of meshing gears is the summation of Hertzian contact energy, bending energy, shear energy, and axial compressive energy that corresponds to Hertzian contact stiffness, bending stiffness, shear stiffness and axial compressive stiffness, respectively [8,11]. Several researchers also considered the fillet foundation deflection in mesh stiffness evaluation [13,27–29]. They all used the fillet foundation deflection equation derived in Ref. [30]. According to the potential energy method [6,7,11], the rectilinear mesh stiffness of a pair of gears can be evaluated as follows:

$$k_l = \begin{cases} \frac{F_1}{\Delta_{11}+\Delta_{21}} + \frac{F_2}{\Delta_{12}+\Delta_{22}}, & \text{double tooth pairs in meshing} \\ \frac{F_1}{\Delta_{11}+\Delta_{21}}, & \text{single tooth pair in meshing} \end{cases}, \tag{1}$$

where F_1 and F_2 are gear mesh forces, and Δ_{11}, Δ_{21}, Δ_{11}, and Δ_{21} are tooth deflections along the action line (see Fig. 1).

Finite element method is another approach to evaluating gear mesh stiffness. Pandya et al. [16] developed a 2-D finite element model to investigate crack effect on the rectilinear mesh stiffness of a gear pair. Song et al. [31] proposed a finite element model for a pair of marine crossed beveloid gears and demonstrated that gear misalignment had little effect on the rectilinear mesh stiffness. Parker et al. [32] developed a combined finite element/contact mechanics model to study non-linear dynamic responses of a pair of spur gears. Later, this model was used to evaluate the mesh stiffness of a planetary gear set [18]. This model reduces the number of finite elements used and enables the mesh stiffness evaluation with practically feasible run time [33].

Wang and Howard [24] discussed several finite element modeling methods for torsional mesh stiffness evaluation. Jia and Howard [23] utilized finite element models to investigate the influence of localised spalling and crack damages on torsional mesh stiffness, respectively. Song et al. [25] applied finite element models to evaluate torsional mesh stiffness of a spur planetary gear set. Cooley et al. [34] discussed the advantages and disadvantages of two mesh stiffness evaluation methods: the average and local slope approaches.

Howard et al. [6] established the relationship between the rectilinear mesh stiffness and the torsional mesh stiffness for gears with assumed rigid gear bodies. Liang et al. [35] used Howard's model [6] to evaluate the mesh stiffness of a pair of spur gears with tooth pitting. A linear finite element model is developed in Ref. [35]. For a pair of gears, we call the smaller one as a pinion and the other one as a gear. Let's fix the pinion gear body and apply a torque T on the body of the gear. The angular displacement of the gear body is denoted by θ (see Fig. 2). The relationship between the rectilinear mesh stiffness and the torsional mesh stiffness is then given as follows [6]: