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Bayesian inference and model comparison for metallic fatigue data

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Highlights

- Several models are calibrated to the 75S-T6 data set by means of the maximum likelihood method.
- Classical measures of fit based on information criteria are used to compare and rank models.
- Bayesian approach is considered to analyze some models under two different a priori scenarios.
- Bayes factor and predictive information criteria are used to compare Bayesian models.

Abstract

In this work, we present a statistical treatment of stress-life (S-N) data drawn from a collection of records of fatigue experiments that were performed on 75S-T6 aluminum alloys. Our main objective is to predict the fatigue life of materials by providing a systematic approach to model calibration, model selection and model ranking with reference to S-N data. To this purpose, we consider fatigue-limit models and random fatigue-limit models that are specially designed to allow the treatment of the run-outs (right-censored data). We first fit the models to the data by maximum likelihood methods and estimate the quantiles of the life distribution of the alloy specimen. To assess the robustness of the estimation of the quantile functions, we obtain bootstrap confidence bands by stratified resampling with respect to the cycle ratio. We then compare and rank the models by classical measures of fit based on information criteria. We also consider a Bayesian approach that provides, under the prior distribution of the model parameters selected by the user, their simulation-based posterior distributions. We implement and apply Bayesian model comparison methods, such as Bayes factor ranking and predictive information criteria based on cross-validation techniques under various a priori scenarios.

MSC: 62N05; 62N01; 62P30; 62F15

Keywords: Metallic fatigue data; Fatigue life prediction; Random fatigue-limit models; Maximum likelihood methods; Bayesian computational techniques for model calibration/ranking; Predictive accuracy for Bayesian models

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1. Introduction

Mechanical and structural components subjected to cyclic loading are susceptible to cumulative damage and eventual failure through an irreversible process called metal fatigue. Prediction of such fatigue through the expected service life of mechanical parts and assemblies is an important objective of numerical simulations used in mechanical and structural engineering practice. Based on such predictions, inspection intervals can be established. The frequency of these inspection intervals bears on the safety and costs of operation [1–3].

The fatigue characteristics of materials are established through fatigue tests performed on coupons, also called dogbone specimens, made of round bars or flat plates. The coupons are designed such that the stress is highest in the gauge section and that it remains substantially constant when the coupon is loaded in the axial direction. In bending and torsion tests, the stress varies linearly over the cross section and is constant in the axial direction, for any fixed point in the cross section.

The number of cycles to failure, the peak stress and the cycle ratio are recorded for each experiment. The cycle ratio is defined as the minimum stress to maximum stress ratio. When an experiment is stopped before the specimen fails, then the test record is marked as a run-out. In some experiments, the specimen may buckle or fail outside of the gauge section. Such experiments are disregarded. State-of-the-art reviews on mechanical fatigue are presented in [1] and [3]. Here, we focus on high-cycle (stress-life) fatigue.

The set of data pairs (S_i, N_i), where S_i is the stress and N_i is the corresponding number of cycles at failure in the ith test, exhibits substantial statistical dispersion. Interpretation and generalization of test data are essential for making risk-informed design decisions. The goal is to find a probability distribution for the fatigue life given data and underlying assumptions. There are many possible phenomenological models which will lead to different results. These results can be derived by different statistical frameworks, among them the frequentist and the Bayesian approaches. Furthermore, there are several ways to judge the results obtained by the use of different models. Various statistical models such as lognormal, extreme value, Weibull and Birnbaum–Saunders distributions have been used for this purpose.

We consider different types of models that contain fatigue limit parameters. Although such models have been widely used (see, for example, [4–7]), there is an ongoing debate concerning the existence of the fatigue limit [8,9]. Some authors use the terms "endurance limit" or "fatigue strength" instead of "fatigue limit" [1,6]. We distinguish between the fatigue limit, which is a physical notion, and the fatigue limit parameter, which is an unknown parameter, expressed in the same scale as the equivalent stress and calibrated for different models. Usually, data support curve fitting up to a certain number of cycles to failure only. Extrapolation beyond that number substantially increases uncertainty. For example, aluminum does not have a fatigue limit, since it will always fail if tested to a sufficient number of cycles. Therefore, the fatigue limit (fatigue strength) of aluminum is reported as the stress level at which the material can survive after a large number of cycles. For the purposes of this paper, the number of cycles can be fixed at 2×10^7, since the available data do not contain substantially larger cycle values.

We employ a classical (likelihood-based) approach to fit and compare the proposed models using the 75S-T6 aluminum sheet specimen data set described in Section 2. Ultimately, we provide an analog Bayesian approach to fit and compare the models. The classical approach provides a point estimation (Maximum Likelihood estimate) for the model parameter θ that lies in the 90% confidence interval if it were repeatedly used with random data from the model for fixed θ. In the Bayesian formulation, no repetition is required and the interval estimation is based on the posterior distribution. Although Ryan used a Bayesian approach to find an optimal design for the random fatigue-limit model [7], we are, to our knowledge, the first to use Bayesian methods to analyze and compare fatigue models.

The remainder of this paper is organized as follows. Section 2 introduces the main characteristics of the fatigue tests conducted at the Battelle Memorial Institute on 85 75S-T6 aluminum sheet specimens by means of a Krouse direct repeated-stress testing machine. The data set with the fatigue test results is available as a csv file in the supplemental material to this paper (see Appendix A). This data set contains run-outs. Section 3 presents classical statistical models of fatigue test results. In Section 3.1, we first consider a classical statistical fitting technique, called logarithmic fit, for illustration purposes only, that does not take into account the presence of run-outs. Subsequently, we introduce fatigue-limit models and random fatigue-limit models, which are both specially designed to fit data in the presence of run-outs. We fit two fatigue-limit models, whose mean value function is same as in the logarithmic fit, with constant and non-constant variance functions, by constructing the corresponding likelihood functions and estimating all the unknown parameters that define the S-N curves by means of the maximum likelihood method. The fatigue limit