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Jacobian projection reduced-order models for dynamic systems with contact nonlinearities

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A B S T R A C T

In structural dynamics, the prediction of the response of systems with localized nonlinearities, such as friction dampers, is of particular interest. This task becomes especially cumbersome when high-resolution finite element models are used. While state-of-the-art techniques such as Craig-Bampton component mode synthesis are employed to generate reduced order models, the interface (nonlinear) degrees of freedom must still be solved in-full. For this reason, a new generation of specialized techniques capable of reducing linear and nonlinear degrees of freedom alike is emerging. This paper proposes a new technique that exploits spatial correlations in the dynamics to compute a reduction basis. The basis is composed of a set of vectors obtained using the Jacobian of partial derivatives of the contact forces with respect to nodal displacements. These basis vectors correspond to specifically chosen boundary conditions at the contacts over one cycle of vibration. The technique is shown to be effective in the reduction of several models studied using multiple harmonics with a coupled static solution.

In addition, this paper addresses another challenge common to all reduction techniques: it presents and validates a novel a posteriori error estimate capable of evaluating the quality of the reduced-order solution without involving a comparison with the full-order solution.

1. Introduction

The dynamics of structures constrained through frictional contacts can be very complex due to the nonlinear nature of dry friction [1]. Often, the response is assumed to be periodic so that harmonic balance methods (HBM) can be used in the frequency domain. Methods to compute the steady-state response can use HBM together with alternating frequency-time (AFT) approaches for systems with friction contacts [2–6].

Spatial reduction techniques like the proper orthogonal decomposition can be used to model the physical DOFs of the structure (i.e. the nodal DOFs of the FE model) as a linear superposition of shape functions [7]. However these techniques require the solution of the full system to be computed at given time instants, and this may be unfeasible for high-resolution FE models. Another approach which also requires the nonlinear system to be solved in advance in order to generate the ROM is described for turbine bladed disks applications in [8].

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Nomenclature

Abbreviations

AFT	Alternating Frequency Time method
CB-CMS	Craig Bampton-Component Mode Synthesis
DoF(s)	Degree(s) Of Freedom
FFT	Fast Fourier Transform
FE	Finite Elements
FO	Full Order
FR	Full Reduction
HBM	Harmonic Balance Method
IFFT	Inverse Fast Fourier Transform
JP	Jacobian Projection
NLR	NonLinear Reducion
RO(M)	Reduced Order (Model)

Vectors and matrices

$\mathbf{D}$	dynamic stiffness matrix
$\mathbf{f}$	vector of forces
$\mathbf{\Phi}$	reduction basis
$\mathbf{g}$	vector of forces in reduced coordinates - FR
$\mathbf{H}$	reduced dynamic stiffness matrix - NLR
$\mathbf{J}$	Jacobian matrix (multi-harmonic)
$\widehat{\mathbf{K}}$	multi-harmonic matrix of partial derivatives of contact forces with respect to displacements
$\mathbf{l}$	vector of forces in reduced coordinates - NLR
$\mathbf{\Lambda_{JC}}$	diagonal matrix of eigenvalues
$\mathbf{M}, \mathbf{C}, \mathbf{K}$	mass, structural damping and stiffness matrices
$\mathbf{M_J}, \mathbf{C_J}, \mathbf{K_J}$	multi-harmonic mass, structural damping and stiffness matrices, harmonics: 0-H
$\mathbf{M_{JC}}, \mathbf{C_{JC}}, \mathbf{K_{JC}}$	multi-harmonic mass, structural damping and stiffness matrices after static condensation, harmonics: 1-H (after static condensation)
$\mathbf{p}$	vector of displacements in reduced coordinates - FR
$\mathbf{\Psi}$	mode used to compute the boundary conditions necessary to the definition of $\mathbf{\Phi}$
$\mathbf{q}$	vector of displacements
$\mathbf{R}$	rotation matrix
$\mathbf{r}$	multi-harmonic vector of residuals
$\mathbf{s}$	vector of displacements in reduced coordinates - NLR
$\mathbf{S}$	diagonal matrix of singular values
$\mathbf{U}, \mathbf{V}$	matrices of left and right singular vectors respectively
$\mathbf{V_{JC}}$	multi-harmonic eigenvectors matrix

Additional subscripts

$_{\mathbf{cond}}$	conditioned
$_{\mathbf{C}}$	contact
$_{\mathbf{E}}$	external
$_{i}$	iteration step
$_{\mathbf{ST}}$	relative to a full stick linear system
$_{*}$	relative to a posteriori error estimate
$_{\mathbf{L}}$	relative to linear DoFs
$_{\mathbf{N}}$	relative to nonlinear DoFs
$_{\mathbf{ROM}}$	relative to the reduced order model

Additional superscripts

^{h}	harmonic index h, both real and imaginary part
^{h,I}	harmonic index h, imaginary part
^{h,R}	harmonic index h, real part
$\overline{()}$	referred to a complex-valued quantity where real and imaginary parts have been split into separate entries of a matrix or vector
^{T}	transpose