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Heuristic and Eulerian interface capturing approaches for shallow water type flow and application to granular flows

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Abstract

Determining the wet–dry boundary and avoiding the related spurious thin-layer problem when solving the depth-averaged shallow-water (SW) equations (or similar granular flow models) remains an outstanding challenge, though it has been the focus of much research effort. In this paper, we introduce the use of level set and phase field based methods to address this issue and related problems. We also propose new heuristic methods to address this problem. We implemented all of these methods in TITAN2D, which is a parallel adaptive mesh refinement toolkit designed for numerical simulation of granular flows. Results of the methods for flow over a simple inclined plane and Colima volcano are used to illustrate the methods. For the inclined plane, we compared the results with experimental data and for Colima volcano they are compared to field data. Our approaches successfully captured the interface of the flow and solved the accuracy and stability problems related to the thin layer problem in SW numerical solution. The comparison of results shows that although all of the methods can be used to address this problem, each of them has its own advantages/disadvantages and methods have to be chosen carefully for each problem.

Keywords: Shallow water flow; Thin layer; Wetting/drying; Phase field; Level set

1. Introduction

Shallow water (SW) flows include a wide range of fluid flows in which the fluid depth is much smaller ($\mathcal{O}(10^{-1})$) than the characteristic length of the fluid body. The shallowness of flow allows us to approximate the variation of state variables in the direction perpendicular to the basal surface by an integrated average [1], which thus reduces a three dimension flow problem into a two-dimensional one. This approximation holds for many geophysical flows and the same conservation equations with minor variations can be used to study different physical situations. Eglit and Sveshnikova [2] modified the depth-averaged Saint Venant equations for water flows to simulate granular snow avalanches and almost a decade later Savage and Hutter [1] popularized these in the modeling of many geophysical mass flows related to landslides, avalanches and debris flows. Since this type of flow has free moving boundaries,

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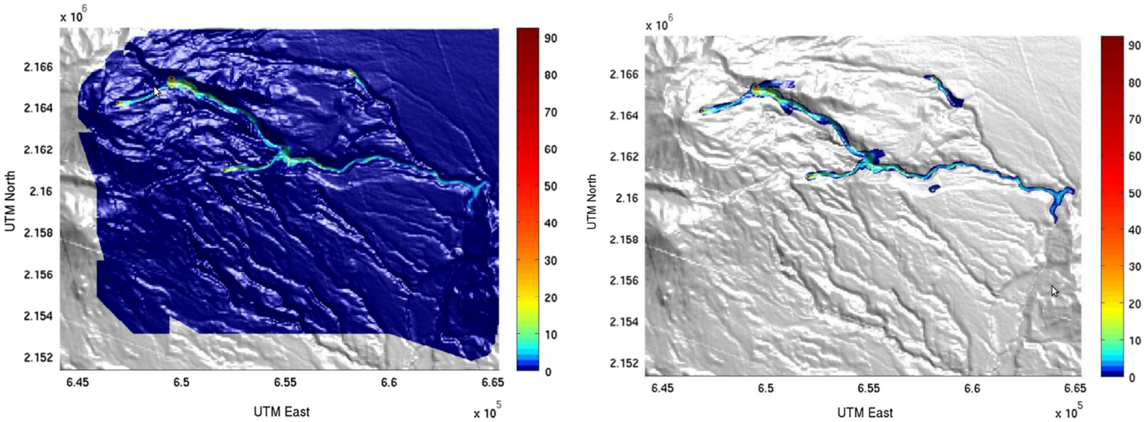

(a) Result without any control. A thin layer of flow covers most of the domain.

(b) Only flow with depth greater than 0.5 m is displayed.

Fig. 1. Thin layer problem in maximum over time flow depth simulation of the 1955 debris flow at Atenquique, Mexico [5].

identifying the location of the flow interface is a critical challenge for successful numerical methods. Furthermore, the governing equations are valid only in the wet areas so we need a strategy to discriminate between wet and dry areas in the numerical simulation. In the SW context, this is usually called the *wetting and drying (WD)* problem. In our previous work [3], we showed the advantages of modifying the speed of waves near the vacuum region based on Toro's approach [4] to mitigate stability concerns. However, this still leads to the formation of a non-physical thin layer in the numerical solution (see Fig. 1 for an illustration). This unphysical thin layer could extend large distances from the realistic main body of the flow, which can cause inaccurate construction of the boundary, loss of conservation or severe numerical instabilities in the numerical solution. Besides the numerical issues, determining probable flow extents through numerical simulation is critical for application of SW equations to geophysical flow. For example, in preparing a hazard map for a volcano or a flood, it is crucial to know the location of the front of the flow to answer basic questions such as—Does the flow reach a specific location? What is the distance of high risk locations from civil infrastructure? The answer of all the above questions is not possible without good information about the interface of the flow along its flow path. A demonstration of possible issues in shown in Fig. 1. This figure displays the numerical simulation of a block of ash flow in Atenquique, a village near the Colima volcano in Mexico, using a SW like model based on a granular flow assumption [3]. The left figure shows the numerical solution of flow height without using any control for the thin layer problem and the right figure shows the same result using a naive control—plotting the regions with flow height $h > 0.5$ m (a threshold deemed too high for hazard analysis). As can be seen in Fig. 1(a), if no control on the numerical solution is used a thin layer of the flow covers a huge part of the domain which causes instability and inaccuracy in the obtained results. To summarize, the following major difficulties arise in numerical solutions of SW flows related to the WD or thin layer problem:

1. *Ambiguous and subjective computation of flow spreads.*
2. *Unphysical fast estimate of the flow speed.* The state variables in the SW equations are momentum, $\{hV_x, hV_y\}$ (product of flow height and velocity). To find flow velocities, $\{V_x, V_y\}$, used during the solution process the momentum (often a small number) must be divided by flow depth, h (also small near the flow boundary), which can cause a large numerical error and results in overly large flow velocities.
3. *Unphysical thin layer (orders of magnitude thinner than a grain of sand).* This results from unphysical speeds and the numerical wicking away of material.
4. *Loss of numerical stability.* Wave-speeds can become infinite at the flow boundary which means that flow equations lose their hyperbolicity at the vacuum state interface.

In addition to scaling issues, correctly identifying interface regions will allow us to construct models for other interesting physics (e.g. entrainment) that happens at the interface.