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# Model updating strategy for structures with localised nonlinearities using frequency response measurements

Xing Wang [a,*], Thomas L. Hill [a], Simon A. Neild [a], Alexander D. Shaw [b], Hamed Haddad Khodaparast [b], Michael I. Friswell [b]

[a] *Department of Mechanical Engineering, University of Bristol, Bristol BS8 1TR, United Kingdom*
[b] *College of Engineering, Swansea University, Swansea SA1 8EN, United Kingdom*

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ABSTRACT

This paper proposes a model updating strategy for localised nonlinear structures. It utilises an initial finite-element (FE) model of the structure and primary harmonic response data taken from low and high amplitude excitations. The underlying linear part of the FE model is first updated using low-amplitude test data with established techniques. Then, using this linear FE model, the nonlinear elements are localised, characterised, and quantified with primary harmonic response data measured under stepped-sine or swept-sine excitations. Finally, the resulting model is validated by comparing the analytical predictions with both the measured responses used in the updating and with additional test data. The proposed strategy is applied to a clamped beam with a nonlinear mechanism and good agreements between the analytical predictions and measured responses are achieved. Discussions on issues of damping estimation and dealing with data from amplitude-varying force input in the updating process are also provided.



## 1. Introduction

Using linear finite-element (FE) models as prototypes to predict structural dynamic behaviours in the design stage is widely accepted in the engineering industry to reduce cost and time. These mathematical models are routinely updated by relaxing weak assumptions or modifying inaccurate parameters after vibration testing campaigns [1,2] to ensure high reliability for load analysis. In practice, many structures are unlikely to behave perfectly linearly during these tests, especially when they respond at large amplitudes. Currently it is common to neglect such nonlinearities as they have marginal effects. However, with the drive towards more efficient and flexible structures, nonlinear dynamic behaviour is inevitably becoming more common. For such structures, linear models no longer achieve high-fidelity predictions and may, in some instances, fail to capture critical dynamic behaviours.

An example of this in aerospace systems is the Cassini Spacecraft, where the longitudinal modal frequency of its payload, the Huygens probe, decreased as forcing level increased. This frequency reduction introduced coupling with the frequency band of high-energy excitation from the launch vehicle and resulted in 50% overload compared to its design requirements [3]. Kerschen et al. [4–6] also observed strong nonlinear behaviours of a wheel elastomer mounting system (WEMS) device during the Smallsat ground test campaign. The piecewise linear WEMS device in the satellite exhibited dynamic behaviours

* Corresponding author.
*E-mail address:* xing.wang@bristol.ac.uk (X. Wang).

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such as jumps and modal interactions that cannot be described using linear models. Göge et al. investigated typical nonlinear phenomena during ground vibration test campaigns on a large aircraft, and demonstrated multiple types of amplitude and/or frequency dependent behaviours [7]. Such nonlinear behaviours may also occur at interfaces when assembling individual sub-components, for example, bilinear stiffness at engine flange joints [8] or wing-pylon joints [9,10]. Along with nonlinearities that are intrinsic to a structure, engineers also intentionally design nonlinear mechanisms to achieve better performance. Examples include employing a nonlinear energy sink to suppress aeroelastic instability [11,12], introducing a nonlinear hinge between wing tips and the main airframe to alleviate gust loads [13], and adding a nonlinear secondary spring to improve the vibration attenuation efficiency of a isolator [14]. These nonlinear mechanisms are typically local such that the number of nonlinear elements is far fewer than the total number of degrees of freedom (DOFs) in the structure; however, they can still result in significant nonlinear behaviours in the global dynamics [11–15].

In practical applications, nonlinear elements are often quite complex to model and accurate parameters are difficult to obtain directly. Thus, tremendous efforts have been devoted to identifying nonlinear elements based on the measured responses of an assembled structure. Kerschen [16] proposed a general nonlinear identification procedure consisting of three main steps (i.e., detection, characterisation and parameter estimation), where the characterisation step relied on the time domain restoring force surface (RFS) method [5] and the parameter estimation step utilises the conditioned reverse path (CRP) method [17] based on random response data. Two comprehensive reviews, including recent developments in this procedure, are documented in references [18] and [19]. Ewins et al. [20,21] proposed a 'Modal Test +' procedure to extend the established linear modal testing techniques for structures with discrete nonlinearities, which also relied on the restoring force surface method and the reverse path method [22] during characterisation and quantification steps, respectively. Recently, the nonlinear normal mode (NNM) based updating strategy [23–26] using measured time series has been presented and applied to the ECL benchmark beam [25] as well as the IMAC XXXII Round Robin benchmark system [26]. Currently, this strategy requires knowledge of the locations and types of nonlinearity present in the structure.

A nonlinear model updating strategy should also verify the updated model and minimise any discrepancy between the predictions and the measured responses [8,21,23–26]. To do this, residuals between measured responses and analytical predictions, based on the identified nonlinear model under the same input, must be extracted and then minimised. While broadband random excitation data, such as that used in the reverse path method, might be used for this, it does raise some issues for large aerospace structures. Firstly, numerical simulations of a nonlinear model under random excitations with a given spectrum generally require time integration schemes. This can be computationally very expensive for a large-scale model during the multiple parameter iterations, as is often the case during updating. Secondly, the aerospace industry typically implements slow swept-sine or stepped-sine excitations to identify structural modal parameters, such as the standard and mandatory procedures adopted by NASA [3], ESA [27] and Airbus [28,29] during ground vibration test campaigns. It would be advantageous for the updating data to be obtained using similar tests for the sake of integration into existing testing practice. Thirdly, and most importantly, the demand to excite the structural modes with sufficient energy is crucial for large-scale aerospace structures. For example, an aircraft should be driven as close as possible to its operational energy levels during ground vibration test campaigns, in order to generate high-quality data to update its FE model or as evidence for the certification process [27,29]. For these applications, random excitation may not be able to excite the structure to sufficiently high amplitudes under actuating limits of current testing equipment [29].

To tackle these challenges, this paper presents a novel model updating strategy for structures with localised nonlinearities. The steps in the nonlinear identification process and numerical simulations are based on the primary harmonic frequency responses that can be obtained using current testing techniques in the aerospace industry; i.e. using stepped-sine or slow swept-sine excitations. It establishes two kinds of residuals to describe the test/analysis discrepancies and avoids computationally expensive time integration schemes by using a direct frequency domain residual minimisation process; this allows rapid iterations to refine the nonlinear FE models and improves the efficiency of updating.

The updating strategy comprises of three main processes: (1) structural testing to obtain the data; (2) linear model updating to construct an underlying linear FE model; (3) nonlinear model updating to localise the nonlinear elements, characterise them, quantify their parameters and then validate the resulting model. These steps are described in detail and demonstrated by using experimental data to update a FE model of a clamped beam with a nonlinear mechanism connected near the tip in Section 3 after the model formulation and residual definition given in Section 2. The resulting model is then assessed using independent experimental data not used in the updating. Following this, Section 4 discusses the issues regarding the estimation of damping and dealing with amplitude-varying force input data in updating. Conclusions are drawn in Section 5.

## 2. Equations and frequency domain residuals

In this section, the form of the equations of motion and the residuals between measured data and predicted responses will be introduced before the updating strategy is proposed in the next section.

### 2.1. Dynamic equation with location matrix

The dynamic equation of an $N$-DOF structure with $\gamma$ inter-connected and/or grounded nonlinear elements can be expressed in a general form as