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Quantifying and managing uncertainty in operational modal analysis

Siu-Kui Au [a,*], James M.W. Brownjohn [b], John E. Mottershead [a]

[a] Center for Engineering Dynamics and Institute for Risk and Uncertainty, University of Liverpool, United Kingdom
[b] College of Engineering, Mathematics and Physical Sciences, University of Exeter, United Kingdom

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ABSTRACT

Operational modal analysis aims at identifying the modal properties (natural frequency, damping, etc.) of a structure using only the (output) vibration response measured under ambient conditions. Highly economical and feasible, it is becoming a common practice in full-scale vibration testing. In the absence of (input) loading information, however, the modal properties have significantly higher uncertainty than their counterparts identified from free or forced vibration (known input) tests. Mastering the relationship between identification uncertainty and test configuration is of great interest to both scientists and engineers, e.g., for achievable precision limits and test planning/budgeting. Addressing this challenge beyond the current state-of-the-art that are mostly concerned with identification algorithms, this work obtains closed form analytical expressions for the identification uncertainty (variance) of modal parameters that fundamentally explains the effect of test configuration. Collectively referred as 'uncertainty laws', these expressions are asymptotically correct for well-separated modes, small damping and long data; and are applicable under non-asymptotic situations. They provide a scientific basis for planning and standardization of ambient vibration tests, where factors such as channel noise, sensor number and location can be quantitatively accounted for. The work is reported comprehensively with verification through synthetic and experimental data (laboratory and field), scientific implications and practical guidelines for planning ambient vibration tests.

1. Introduction

Operational modal analysis (OMA) aims at identifying the modal properties (natural frequency, damping ratio, mode shape, etc) of a constructed structure using only the (output) vibration response (acceleration, velocity, etc) [1–3]. The (input) excitation to the structure is not measured but is assumed to be broadband random. This allows vibration data to be collected when the structure is in its working or 'operational' condition without much intervention. This implies significant economy in implementation, which to a large extent has contributed to the increasing popularity of OMA in practical applications [4–7].

In the absence of loading information, the identification uncertainty of modal parameters from ambient vibration data is significantly higher than that in free vibration or forced vibration tests. This is complicated by variability due to modeling errors regarding the stationary or broadband nature of loading, and the effects of structural/environmental changes

* Corresponding author.
E-mail address: siukuiau@liverpool.ac.uk (S.-K. Au).

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[8–10]. Uncertainty quantification and quality control on the identified modal properties therefore become especially relevant. From a scientific point of view, it is of interest to know what factors the identification uncertainty depends on and what the relationship is. For planning or specification purposes, it is desirable to have an assessment of the identification uncertainty for a given test configuration. For example, how long should the data be? How many sensors are required? Should better sensors be used? These are long-standing issues that have presented challenges to researchers and practitioners [11–14].

A Bayesian approach provides a fundamental basis for extracting the information contained in the data for inferring the parameters of interest in a manner consistent with probability and modeling assumptions [15–17]. In OMA this has recently been materialized and put into practice, where making inference based on the 'raw' FFT (i.e., no filtering, windowing, etc.) within a selected frequency band is found to yield a computationally efficient method whose modeling assumptions are robust to applications. See [18] for the first formulation, [19] for a recent review and [20–24] for examples of recent applications. In a Bayesian context, identification results are encapsulated in the joint 'posterior' (i.e., given data) distribution of the modal parameters. With sufficient data often encountered in applications, the posterior distribution has a single peak and it can be approximated by a Gaussian distribution. The mean of the Gaussian distribution gives the posterior most probable value (MPV) of the modal parameters, while the covariance matrix reflects their remaining identification uncertainty. In a non-Bayesian, or 'frequentist' context, identification uncertainty has been defined as the ensemble variance of estimates over repeated experiments. Methods of calculation based on perturbation have been developed in [25–27] for time-domain state-space models. See also [28] that investigated empirically the effects of various sources on identification results.

Being able to calculate the identification uncertainty for a given set of data alone does not provide much insight about how it depends on test configuration. Due to complexity of the problem, the exact dependence is expected to be complicated and is unlikely to be described in a closed-form explicit manner. Motivated by observations on the identification uncertainty of modal parameters in terms of their posterior c.o.v. (coefficient of variation = standard deviation/mean) monitored during typhoons, an asymptotic analysis has been performed for the posterior covariance matrix [29]. Focusing on well-separated modes, the study yielded closed-form expressions for the leading (zeroth) order of the posterior c.o.v. under the asymptotic condition of small damping and long data duration. The results were collectively referred as 'uncertainty laws', analogous to the laws of large numbers in statistics. They were found to be remarkably simple and insightful.

The theory of uncertainty laws motivated the definition of the 'modal signal-to-noise (s/n) ratio' as the PSD (power spectral density) ratio of the modal response to noise at the natural frequency. This was found to be the only parameter in the uncertainty laws that reflects test configuration attributes such as instrument noise, the number of sensors and their locations. However, the leading (zeroth) order of the uncertainty laws obtained so far does not depend on the modal s/n ratio. In this sense the zeroth order expression gives the 'achievable precision limit' when the modal s/n ratio is infinite. The objective of this work is to further capture the effect of the modal s/n ratio in the uncertainty laws so that test configuration can be quantified for planning or standardizing ambient vibration tests. To achieve this objective, we perform a first order asymptotic analysis of the posterior c.o.v.s, leading to 'first order uncertainty laws'.

This work is organized as follow. We first give a short overview of the Bayesian framework for OMA, based on which the uncertainty laws were derived. The zeroth order laws will then be reviewed. The key results of the first order laws will be summarized, followed by an outline of derivation with details referred to the appendix. The first order laws will be verified and their approximation under non-asymptotic conditions will be investigated using synthetic data and experimental data. Implications and applications of the uncertainty laws for planning ambient vibration tests will also be discussed.

2. Bayesian framework

Let the acceleration time history at n measured DOFs of a structure be $\{\hat{\ddot{\mathbf{x}}}_j \in R^n\}_{j=0}^{N-1}$ and abbreviated as $\{\hat{\ddot{\mathbf{x}}}_j\}$, where N is the number of samples per data channel. The (scaled one-sided) FFT of $\{\hat{\ddot{\mathbf{x}}}_j\}$ is the complex-valued sequence $\{\mathcal{F}_k \in C^n\}_{k=0}^{N-1}$ where

$$\mathcal{F}_k = \sqrt{\frac{2\Delta t}{N}}\sum_{j=0}^{N-1}\hat{\ddot{\mathbf{x}}}_j e^{-2\pi \mathbf{i} jk/N} \tag{1}$$

$\mathbf{i}^2 = -1$ and Δt is the sampling interval. For a given k, the FFT $\mathcal{F}_k$ corresponds to the frequency $\mathrm{f}_k = k/N\Delta t$, up to the Nyquist frequency.

As in the conventional setting, consider a classically damped mode well-separated from other modes. It is identified using only the $\mathcal{F}_k$ s on a selected band near the natural frequency. In the band it is assumed that $\mathcal{F}_k = \mathbf{\Phi}\ddot{\eta}_k + \boldsymbol{\varepsilon}_k$ where $\mathbf{\Phi} \in R^n$ is the mode shape confined to the measured DOFs (scaled to have unit Euclidean norm, i.e., $||\mathbf{\Phi}||^2 = \mathbf{\Phi}^T\mathbf{\Phi} = 1$); $\boldsymbol{\varepsilon}_k \in C^n$ is the FFT of channel noise; and $\ddot{\eta}_k \in C$ is the FFT of the modal acceleration response whose time domain counterpart satisfies $\ddot{\eta}(t) + 2\zeta\omega\dot{\eta}(t) + \omega^2\eta(t) = p(t)$. Here $\omega = 2\pi f$ (rad/s), f is the natural frequency (in Hz), ζ is the damping ratio and $p(t)$ is the modal force. The modal force and channel noise are assumed to have a constant PSD within the selected band, denoted respectively by S and S_e. In the above context, the set of modal parameters to be identified is $\boldsymbol{\theta} = \{f, \zeta, S, S_e, \mathbf{\Phi}\}$.

Let $\{\mathcal{F}_k\}$ denote the collection of FFT data within the selected band. Using Bayes' Theorem with a uniform prior distribution for $\boldsymbol{\theta}$, the posterior PDF can be written as $p(\boldsymbol{\theta}|\{\mathcal{F}_k\}) \propto \exp[-L(\boldsymbol{\theta})]$ where