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Vibratory synchronization transmission of a cylindrical roller in a vibrating mechanical system excited by two exciters

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ABSTRACT

In present work vibratory synchronization transmission (VST) of a cylindrical roller with dry friction in a vibrating mechanical system excited by two exciters, is studied. Using the average method, the criterion of implementing synchronization of two exciters and that of ensuring VST of a roller, are achieved. The criterion of stability of the synchronous states satisfies the Routh-Hurwitz principle. The influences of the structural parameters of the system to synchronization and stability, are discussed numerically, which can be served as the theoretical foundation for engineering designs. An experiment is carried out, which approximately verify the validity of the theoretical and numerical results, as well as the feasibility of the method used. Utilizing the VST theory of a roller, some types of vibrating crushing or grinding equipments, etc., can be designed.

1. Introduction

Synchronization phenomena is everywhere, such as synchronization in neuronal electrical activities, coupled self-sustained electromechanical devices and populations of globally coupled oscillators [1–4]. In light of the origin of the synchronization phenomena, we should proceed from the observation of Huygens [5]. Recently synchronization of pendula or clocks has been appealed to the interests of many researchers, some representatives of which, can be seen in Refs. [6–8].

In vibration utilization engineering, synchronization of exciters, caused by vibration, has been also given a great deal of attentions. Its theoretical explanation is given firstly by Blekhman [9–13] and applied it in engineering successfully, later who gave the definition of synchronization from the point of views of kinematics and dynamics in 1997, and further stressed it in 2002. According to the definition of Blekhman, synchronization in its most general interpretation means correlated or corresponding in-time behavior of two or more processes. Such behavior of interconnection system can be seen as mapping specific functions of system state variables, which may be scalar, vector or matrix, in other words, it may be a constant or time-varying. Wen [14–17], simplified the mathematical investigating process of synchronization theory, by combining the differential equations of two identical exciters into that of their phase difference. Additionally, times frequency synchronization in a nonlinear vibrating system, including the high order harmonic and the sub harmonic frequencies, such as 2 times frequency, 3 times frequency and n times frequency, is studied by Wen. Balthazar [18,19] gave some short comments of synchronization of exciters on a flexible portal frame structure. Controlled synchronization of three exciters was given by adaptive sliding mode control algorithm [20], as well as speed synchronization control for integrated automotive motor–transmission powertrain system [21].

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Nomenclature

f_i	damping coefficient of axes of the induction motor i, $i = 1, 2$
f_j	damping constant of a vibrating system in j – direction, $j = x, y, \psi$
f_R	friction coefficient between a roller and the inwall of the cavity
g	acceleration of gravity
I_R	moment of inertia of a roller about itself spin axes o_R
J	moment of inertia of the total vibrating system about its mass center, $J = Ml_e^2 = J_m + J_R + 2m_0\left(r^2 + l_0^2\right)$
J_i	moment of inertia of the exciter i $(i = 1, 2)$, $J_1 = J_2 = m_0r^2$
J_m	moment of inertia of the rigid frame
J_R	moment of inertia of a roller, $J_R = m_Rr_R^2 + I_R$
k_i	stiffness of the vibrating system in i- direction, $i = x, y$
k_ψ	stiffness of the vibrating system in ψ-direction, $k_\psi = \left(k_yl_x^2 + k_xl_y^2\right)/2$
$\mathbf{K}_j$	stiffness matrix of the spring j, $j = 1, 2, 3, 4$, $\mathbf{K}_1 = \mathbf{K}_3 = \mathrm{diag}(k_x/2, 0)$, $\mathbf{K}_2 = \mathbf{K}_4 = \mathrm{diag}(0, k_y/2)$
l_e	equivalent rotational radius of the total vibrating system about its mass center, $l_e = \sqrt{J/M}$
l_0	distance between the rotary center of each exciter and the mass center of a rigid frame
m_0	mass of the standard exciter
m_i	mass of the exciter i $(i = 1, 2)$, $m_1 = m_2 = m_0$
m_R	mass of a cylindrical roller
m	mass of the rigid frame
M	mass of the total vibrating system, $M = m + m_1 + m_2 + m_R$
r	eccentric radius of each exciter (it is here assumed that eccentric radiuses of two exciter are identical)
r_R	rotational radius of a roller rotating about the center of the cavity along the inwall
r_l	$r_l = l_0/l_e$
r_m	mass ratio of the standard exciter to the total vibrating system, $r_m = m_0/M$
r_{mr}	mass ratio of a roller to the total vibrating system, $r_{mr} = m_R/M$
T	kinetic energy of the vibrating system, $T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}J_m\dot{\psi}^2 + \frac{1}{2}\sum_{i=1,2}J_i\dot{\varphi}_i^2 + \frac{1}{2}(J_R - I_R)\dot{\varphi}_R^2 + \frac{1}{2}I_R\dot{\theta}_R^2 + \frac{1}{2}\sum_{i=1,2,R}m_i\dot{\mathbf{x}}_i^T\dot{\mathbf{x}}_i$
T_{e0i}	electromagnetic torque of the induction motor operating steadily at the angular velocity ω_{m0}, $i = 1, 2$
T_{ei}	electromagnetic torque of the induction motor i, $i = 1, 2$
T_u	energy of the standard exciter, $T_u = m_0r^2\omega_{m0}^2/2$
V	potential energy of the system, $V = \frac{1}{2}\sum_{j=1}^{4}(\mathbf{X}_{kj} - \mathbf{X}_{k0j})^T\mathbf{K}_j(\mathbf{X}_{kj} - \mathbf{X}_{k0j})$.
$\mathbf{x}_o$	displacement vector of the mass center of the rigid frame m, $\mathbf{x}_o = \{x, y\}^T$.
$\mathbf{x}_i$	coordinates of two exciters and a roller in the reference frame oxy, $\mathbf{x}_i = \mathbf{x}_o + \mathbf{R}\mathbf{x}_i''$, $\mathbf{R} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}$, $i = 1, 2, R$
$\mathbf{x}_i''$	coordinates of two exciters and a roller in the reference frame $o'x''y''$, $i = 1, 2, R$, $\mathbf{x}_1'' = \begin{Bmatrix} l_0 + r\cos\varphi_i \\ r\sin\varphi_i \end{Bmatrix}$, $\mathbf{x}_2'' = \begin{Bmatrix} -l_0 + r\cos\varphi_i \\ r\sin\varphi_i \end{Bmatrix}$, $\mathbf{x}_R'' = \begin{Bmatrix} r_R\cos\varphi_R \\ r_R\sin\varphi_R \end{Bmatrix}$
$\mathbf{X}_{k0j}$	initial position vector of the point that the spring j is connected to the rigid frame, $j = 1, 2, 3, 4$, $\mathbf{X}_{k01} = \{-l_x, 0\}^T$, $\mathbf{X}_{k02} = \{0, l_y\}^T$, $\mathbf{X}_{k03} = \{l_x, 0\}^T$, $\mathbf{X}_{k04} = \{0, -l_y\}^T$
$\mathbf{X}_{kj}$	position vector of the point $\mathbf{X}_{k0j}$ during the running of the vibrating system, $j = 1, 2, 3, 4$
α_1	phase difference between two exciters, $\varphi_1 - \varphi_2 = 2\alpha_1$
α_2	phase difference between exciter 2 and a roller, $\varphi_2 - \varphi_R = 2\alpha_2$
$\pi - \gamma_i$	phase angle of the vibrating system in i-direction, $\gamma_i = \arctan\frac{2\xi_{ni}(\omega_{ni}/\omega_{m0})}{1-(\omega_{ni}/\omega_{m0})^2}$, $i = x, y, \psi$
θ_R	phase of a roller about itself spin axes o_R
ξ_{ni}	critical damping ratio of the vibrating system in i-direction, $i = x, y, \psi$
φ	average phase among two exciters and a roller, $\varphi = (\varphi_1 + \varphi_2 + \varphi_R)/3$
φ_i	phase of the exciter i, $i = 1, 2$
φ_R	phase of a roller
ω_{m0}	synchronous angular velocity of two exciters and a roller in the steady state
ω_{ni}	natural frequency of the vibrating system in i-direction, $\omega_{ni} = \sqrt{k_i/M}$, $i = x, y$
$\omega_{n\psi}$	natural frequency of the vibrating system in ψ-direction, $\omega_{n\psi} = \sqrt{k_\psi/J}$
$(\dot{\bullet})$	$\mathrm{d}\bullet/\mathrm{d}t$
$(\ddot{\bullet})$	$\mathrm{d}^2\bullet/\mathrm{d}t^2$

The key factor to implement synchronization of exciters in a vibrating system, is the coupling effect existing among exciters, which is capable of adjusting the energy balance of exciters to reach synchronization. Using the average method of small parameters, authors have given some investigations on synchronization theory of two or three non-identical exciters, in a super-resonant vibrating system with small damping (i.e., critical damping ratio of the system, denoted by ξ, is very small, $\xi \leqslant 0.07$) [22–25]. In studying synchronization theory of exciters above, there exists a particularly phenomenon, i.e., VST. The