Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 304 (2016) 378–407
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Isogeometric triangular Bernstein–Bézier discretizations: Automatic mesh generation and geometrically exact finite element analysis

Luke Engvall*, John A. Evans

Aerospace Engineering Sciences, University of Colorado at Boulder, United States

Received 14 September 2015; received in revised form 2 February 2016; accepted 5 February 2016
Available online 3 March 2016

Abstract

Isogeometric analysis (IGA) was introduced as a way to bypass the design-to-analysis bottleneck inherent in the traditional Computer Aided Design (CAD) through Finite Element Analysis (FEA) paradigm. However, an outstanding problem in the field of IGA is that of surface-to-volume parameterization. In CAD packages, solid objects are represented by a collection of NURBS or T-spline bounding surfaces, but to perform engineering analysis on real world problems, we must find a way to parameterize the volumes of these objects as well. This has proven to be difficult using traditional IGA, as the tensor-product nature of trivariate NURBS and T-splines limit their ability to create analysis suitable parameterizations of arbitrarily complex volumes.

To overcome the limitations of trivariate NURBS and T-splines, we propose the use of rational Bernstein–Bézier tetrahedra to create analysis suitable volumetric parameterizations for isogeometric analysis. In this paper, which is part one of a two part series, we present the methodology for discretizing two dimensional geometries using rational Bernstein–Bézier triangles. In addition to presenting finite element analysis methodologies based on rational Bernstein–Bézier triangles, we also introduce two new mesh generation strategies for automatically creating high quality, geometrically exact curvilinear meshes. We assess the quality of our mesh generation schemes using a suite of challenging two-dimensional geometries, and we verify the accuracy of our proposed numerical discretization approach using the method of manufactured solutions.

Keywords: Isogeometric analysis; Mesh generation; Volume parameterization

1. Introduction

In the product design cycle, designers and engineers will often iterate over many design permutations. A product is designed, some form of engineering analysis is performed, and the analysis in turn informs the next design iteration. With the ever increasing computational resources available to the modern engineer, Computer Aided Design (CAD) and Finite Element Analysis (FEA) are becoming more and more prevalent in this design cycle.

Ideally, engineers should be able to easily leverage these tools in tandem to converge on optimal designs. That is, there should be easy and seamless transfer of information between CAD and FEA. However, in the traditional CAD

* Corresponding author.
E-mail address: engvall@colorado.edu (L. Engvall).

http://dx.doi.org/10.1016/j.cma.2016.02.012
0045-7825/

to FEA paradigm, this is not the case. This disconnect between CAD and FEA leads to what has been termed the Design-to-Analysis bottleneck.

Isogeometric Analysis (IGA) was conceived in 2005 as a solution to this very problem [1]. By using the functions native to CAD packages (NURBS) as the basis for finite element analysis, the tight link between CAD and FEA is established. IGA has been applied to many problems of engineering interest with promising results. However, traditional isogeometric analysis still suffers limitations in representing arbitrary volumes. This is because CAD packages represent solid objects using a collection of NURBS and/or T-spline bounding surfaces, but there is no numerical parameterization of the interior domain. Indeed, this proves to be a major drawback to isogeometric analysis, as its ability to remove the design-to-analysis bottleneck is inhibited for many problems of real engineering interest, including problems arising in acoustics, electromagnetics, and fluid dynamics.

In the field of isogeometric analysis, this aforementioned issue is referred to as "surface-to-volume parameterization". Roughly speaking, the goals of surface-to-volume parameterization are as follows:

Given a NURBS or T-spline surface from CAD, automatically parameterize the corresponding volume such that:

1. *The parameterization exactly matches the geometry.*
2. *The parameterization is analysis suitable.*[1]
3. *The parameterization is defined by piecewise polynomial or rational basis functions that satisfy linear independence, partition of unity, positivity, and higher-order continuity.*

The three properties listed above are necessary for a parameterization to be suitable for isogeometric analysis. The first property is obviously necessary to ensure that the exact geometry is employed in analysis, while the final two properties ensure that an analysis framework built on the parameterization is well-posed [2], robust [3] and exhibits optimal convergence rates [4].

The surface-to-volume problem is non-trivial, and there exists a large body of work on attempts to solve this specific problem using trivariate B-spline, NURBS, and T-spline parameterizations. However, it is the opinion of these authors that none of the previous work successfully addresses all of the goals listed above. Presented below is a short summary of the state of the art, as well as a short discussion of the advantages and disadvantages of each of the methods with respect to the goals listed above.

Early work into the problem of surface-to-volume parameterization focused on generating trivariate B-spline and NURBS discretizations. One approach, proposed by Aigner et al., is to use swept volumes to create trivariate NURBS parameterizations, which has been shown to work well for a suite of relatively simple geometries, such as low-genus topologies, and geometries with small variations in cross-sectional profiles [5]. However, this method requires sweeping curves to be defined, and is therefore not automatic. Additionally, there is no study of analysis suitability of the parameterization, and the method is limited to the restrictive class of geometries described above. Another method, proposed by Martin et al. [6], attempts to fit trivariate B-splines to tetrahedral meshes using harmonic functions. However, this method is not fully automatic, and is also not truly isogeometric as it uses trivariate B-splines to parameterize the volume, and therefore cannot exactly represent certain geometries of engineering interest, such as conic sections. Finally, it is worth noting that any method for volume parameterization using trivariate NURBS or B-splines suffers from the same limitations as structured hexahedral mesh generation for classical FEM, and are often unable to automatically generate analysis suitable meshes of complex geometries.

Work has also been done to develop volumetric parameterizations using T-splines, which allow for local refinement via T-junctions in the parameterization. A notable example is the meccano method proposed by Escobar et al. [7]. This method shows considerable promise, but the authors acknowledge further work needs to be done to enable automatic mesh generation of arbitrary genus solids, as well as to ensure analysis suitability of meshes.

Many methods for volume parameterization have also been proposed by the Zhang research group. Wang et al. proposed a method for T-spline parameterizations from boundary triangulations [8]. This method has been successful in automatically parameterizing a wide variety of complex volumes, however still suffers from poor mesh quality in some cases. Liu et al. proposed a method for T-spline construction using Boolean operations [9], but it has been shown that while this method can automatically parameterize many complex volumes, it is unable to

[1] We say a parameterization is analysis suitable if it is both invertible and well-conditioned. A parameterization is invertible if the element-wise Jacobian is bounded from both above and below. A parameterization is well-conditioned if it preserves optimal convergence rates. See Appendix A.4 for more details.