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Discrete frequency slice wavelet transform

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ABSTRACT

This paper introduces a new kind of Time-Frequency Representation (TFR) method called Discrete Frequency Slice Wavelet Transform (DFSWT). It is an improved version of Frequency Slice Wavelet Transform (FSWT). The previous researches on FSWT show that it is a new efficient TFR in an easy way without strict limitation as traditional wavelet theory. DFSWT as well as FSWT are defined directly in frequency domain, and still keep its properties in time-frequency domain as FSWT decomposition, reconstruction and filter design, etc. However, the original signal is decomposed and reconstructed on a Chosen Frequency Domains (CFD) as need of application. CFD means that the decomposition and reconstruction are not completed on all frequency components. At first, it is important to discuss the necessary condition of CFD to reconstruct the original signal. And then based on norm l_2, an optimization algorithm is introduced to reconstruct the original signal even accurately. Finally, for a test example, the TFR analysis of a real life signal is shown. Some conclusions are drawn that the concept of CFD is very useful to application, and the DFSWT can become a simple and easy tool of TFR method, and also provide a new idea of low speed sampling of high frequency signal in applications.

1. Introduction

1.1. Background

A great deal of signals obtained from many fields need to be detected and analyzed, for example, the biomedical signals in clinical monitoring, a variant of vibration signals in mechanic system, the response signals in building or bridge monitoring system, etc. Time-frequency (T-F) analysis has been used successfully to characterize these signals due to the fact that the local properties of a signal in time and frequency domains include many information and features [1,2]. Until now, many T-F methods can be used [3,4]. The oldest and simplest one is short-time Fourier transform (STFT), and the common spectrogram can provide a good insight of a signal. To compare with STFT, a Wigner-Ville distribution (WVD) can provide better location for a selected chirp signal. The continuous wavelet transform (CWT) could also present the similar results [5]. On reviewing the characteristics and dilemmas of STFT, WVD, and CWT, Yan et al. [5,6] proposed FSWT for TFR. In fact, FSWT is a significant extension of the STFT in the frequency domain, and the traditional CWT is also an extension in the time domain, and the previous researches [5–7] show that FSWT is a new efficient TFR tool in an easy way without the strict limitation of wavelet

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theory as CWT. The accuracy of damping identification [7] is improved by using FSWT for transient vibration response analysis, and the applications of FSWT can be found in [5,7,8].

1.2. Reviewing on FSWT method

For any $f(t) \in L^2(R)$, the Fourier transformation of $p(t)$ exists, and the **FSWT** can be defined as [6]:

$$W_f(t,\omega,\sigma) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{f}(u)\hat{p}^*\left(\frac{u-\omega}{\sigma}\right)e^{iut}du \tag{1}$$

where, the scale $\sigma \neq 0$ is a constant or a function of ω and t, and the star '$*$' means the conjugate of a function (the following is same). Here we call ω and t as the observed frequency and time, and u the assessed frequency. $\hat{p}(\omega)$ is also called frequency slice function (FSF). The general wavelet [5,6] is a 'microscope' in time domain, but here FSWT is a 'microscope' in frequency domain, and also this transform is called a wavelet transform in frequency domain.

Here, R denotes the set of real numbers. $L^2(R)$ denotes the vectors space of measurable, square-integral one-dimensional functions $f(x)$. Fourier transformation (FT) is expressed by function $f(x) \in L^2(R)$.

$$F\{f\} : \hat{f}(\omega) = \int_{-\infty}^{\infty} f(\tau))e^{-i\omega\tau}d\tau$$

Fourier inverse transformation:

$$F^{-1}\{\hat{f}\} : f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega t}d\omega$$

Theorem 1. [6], if the $\hat{p}(\omega)$ satisfies $\hat{p}(0) = 1$, then the original signal $f(t)$ can be reconstructed by

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_f(\tau,\omega,\sigma)e^{i\omega(t-\tau)}d\tau d\omega \tag{2}$$

From [6], more details of reconstructing methods can be found, here only the Theorem 1 is listed to analyze. Yan et al. [6] have summarized that comparing with traditional wavelet transform FSWT has many advantages. As an important result of Theorem 1, it is significant to note that the reconstructing process is independent with the FSF and its scale σ. For any non-constant scale σ in Eq. (1), the original signal can be reconstructed by Eq. (2) whose computation process is not related with FSF $p(t)$ or $\hat{p}(\omega)$ directly. Therefore, if the condition $\hat{p}(0) = 1$ always remains unchanged in computation of FSWT, more conclusions can be obtained as bellows:

(1) σ in Eq. (1) is able to fit a specific signal in computation if necessary.
(2) The FSF $\hat{p}(\omega)$ can be changed in computation as need.
(3) The truncation of FSF $\hat{p}(\omega)$ does not bring any error for reconstructing the original signal by Eq. (2).

These conclusions mean that whether FSF or scale σ can be chosen dynamically as application requirements. Notably, the general wavelet does not satisfy this kind of dynamic wavelet base function or dynamic scale, because its reconstruction equation must depend on them. Otherwise it is impossible to reconstruct the original signal. Therefore, FSWT provides a smart tool to control the decomposition and reconstruction more easily than traditional wavelet [6].

1.3. A question

However, to reconstruct the original signal accurately by Eq. (2), FSWT as similar with CWT, STFT, and WVD, will cost huge computation time and memories for the completed decomposition of a signal on its all frequency components or the observed scales, and generally, the computation cost become very high and even impossible for general engineering application, especially for high sampling rate with long time series signal. This dilemma is a big trouble to application. Hence, focusing on solving the key problem, in this study, a novel discrete FSWT that can reconstruct original signal accurately will be proposed in the following.

1.4. Objective

Discrete wavelet transform (DWT) have been also successfully applied in many fields [9,10], DWT decomposes the measured signals into mono-component signals with fixed center frequency, so the center frequency cannot be chosen as need. In general, these mono-component signals are not actual signals, and that, the DWT decomposition is not redundant, some investigations of DWT can be found in [5,7]. However, all of the TFR tools such as FSWT, CWT, STFT and WVD are always redundant. Therefore, to reduce a great deal of calculation by reducing re-sampling in time or frequency domain is possible.