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A history of cepstrum analysis and its application to mechanical problems

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ABSTRACT

It is not widely realised that the first paper on cepstrum analysis was published two years before the FFT algorithm, despite having Tukey as a common author, and its definition was such that it was not reversible even to the log spectrum. After publication of the FFT in 1965, the cepstrum was redefined so as to be reversible to the log spectrum, and shortly afterwards Oppenheim and Schafer defined the "complex cepstrum", which was reversible to the time domain. They also derived the analytical form of the complex cepstrum of a transfer function in terms of its poles and zeros. The cepstrum had been used in speech analysis for determining voice pitch (by accurately measuring the harmonic spacing), but also for separating the formants (transfer function of the vocal tract) from voiced and unvoiced sources, and this led quite early to similar applications in mechanics. The first was to gear diagnostics (Randall), where the cepstrum greatly simplified the interpretation of the sideband families associated with local faults in gears, and the second was to extraction of diesel engine cylinder pressure signals from acoustic response measurements (Lyon and Ordubadi). Later Polydoros defined the differential cepstrum, which had an analytical form similar to the impulse response function, and Gao and Randall used this and the complex cepstrum in the application of cepstrum analysis to modal analysis of mechanical structures. Antoni proposed the mean differential cepstrum, which gave a smoothed result. The cepstrum can be applied to MIMO systems if at least one SIMO response can be separated, and a number of blind source separation techniques have been proposed for this. Most recently it has been shown that even though it is not possible to apply the complex cepstrum to stationary signals, it is possible to use the real cepstrum to edit their (log) amplitude spectrum, and combine this with the original phase to obtain edited time signals. This has already been used for a wide range of mechanical applications. A very powerful processing tool is an exponential "lifter" (window) applied to the cepstrum, which is shown to extract the modal part of the response (with a small extra damping of each mode corresponding to the window). This can then be used to repress or enhance the modal information in the response according to the application.

1. Introduction

The first paper on cepstrum analysis [1] defined it as "the power spectrum of the logarithm of the power spectrum". The original application was to the detection of echoes in seismic signals, where it was shown to be greatly superior to the autocorrelation function, because it was insensitive to the colour of the signal. This purely diagnostic application did not require

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returning to the log spectrum, and the reason for the definition was presumably that software for producing power spectra was readily available. Even though [1] had one of the authors of the FFT algorithm (John Tukey) as a co-author it was published two years before the FFT algorithm [2] and thus the potential of the latter was not yet realised. Tukey himself in [1] writes "Although spectral techniques, involving second-degree operations, are now quite familiar, the apparently simpler first-degree Fourier techniques are less well known" (he then refers to a paper of which he is the sole author, which confirms that he made the remark). Thus, the original cepstrum was not reversible even to the log spectrum, and so even though the word and concept of a "lifter" (a filter in the cepstrum) was defined in the original paper, this was enacted as a convolutive filter applied to the log spectrum rather than a window in the cepstrum. The diagnostic application was also satisfactory for determining the voice pitch of voiced speech [3], so speech analysis was one of the earliest applications, much of the development being done at Bell Telephone Labs.

The early history of the development of the cepstrum was recently published by the two pioneers Oppenheim and Schafer [4], but their paper primarily covers applications in speech analysis, communications, seismology and geophysics, and no mechanical applications are given. They describe how Oppenheim's PhD dissertation at MIT [5] introduced the concept of "homomorphic systems", where nonlinear relationships could be converted into linear ones to allow linear filtering in the transform domain. Typical examples were conversion of multiplication into addition by the log operation, and conversion of convolution by first applying the Fourier transform (to convert the convolution to a multiplication) followed by the logarithmic conversion. It was a small further step to perform a (linear) inverse Fourier transform on the log spectrum to obtain a new type of cepstrum, and this was the basis of Schafer's PhD dissertation at MIT [6]. By retaining the phase in all operations, the "complex cepstrum" was defined as the inverse Fourier transform of the complex logarithm of the complex spectrum, and this was thus reversible to the time domain. Oppenheim and Schafer collaborated on a book [7], in which a number of further developments and applications of the cepstrum were reported, in particular the analytical form of the cepstrum for sampled signals, where the Fourier transform could be replaced by the z-transform.

2. Basic relationships and definitions

2.1. Formulations

The original definition of the (power) cepstrum was:

$$C_p(\tau) = |\Im\{\log(F_{xx}(f))\}|^2 \tag{1}$$

where $F_{xx}(f)$ is a power spectrum, which can be an averaged power spectrum or the amplitude squared spectrum of a single record.

The definition of the complex cepstrum is:

$$C_c(\tau) = \Im^{-1}\{\log(F(f))\} = \Im^{-1}\{\ln(A(f)) + j\phi(f)\} \tag{2}$$

where

$$F(f) = \Im\{f(t)\} = A(f)e^{j\phi(f)} \tag{3}$$

in terms of the amplitude and phase of the spectrum. It is worth noting that the "complex cepstrum" is real, despite its name, because the log amplitude of the spectrum is even, and the phase spectrum is odd.

The new power cepstrum is given by:

$$C_p(\tau) = \Im^{-1}\{\log(F_{xx}(f))\} \tag{4}$$

which for the spectrum of a single record (as in (3)) can be expressed as:

$$C_p(\tau) = \Im^{-1}\{2\ln(A(f))\} \tag{5}$$

The so-called real cepstrum is obtained by setting the phase to zero in Eq. (2):

$$C_r(\tau) = \Im^{-1}\{\ln(A(f))\} \tag{6}$$

which is seen to be simply a scaled version of (5).

It should be noted that the complex cepstrum requires the phase $\phi(f)$ to be unwrapped to a continuous function of frequency, and this places a limit on its application. It cannot be used for example with stationary signals, made up of discrete frequency components (where the phase is undefined at intermediate frequencies) and stationary random components (where the phase is random). This point is taken up later. It can only be used with well-behaved functions such as impulse responses, where the phase is well-defined (and often related to the log amplitude). The unwrapping can usually be accomplished using an algorithm such as UNWRAP in Matlab®, although the latter may give errors where the slope of the phase function is steep. This problem can usually be resolved by repeatedly interpolating the spectra by factors of 2 until the same result is obtained with successive steps. The penultimate step can then be used. Errors in unwrapping can also occur when the amplitude is very small, so that noise affects the phase estimates.