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Estimation of geometric properties of three-component signals for system monitoring

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A B S T R A C T

Most methods for condition monitoring are based on the analysis and characterization of physical quantities that are three-dimensional in nature. Plotted in a three-dimensional Euclidian space as a function of time, such quantities follow a trajectory whose geometric characteristics are representative of the state of the monitored system. Usual condition monitoring techniques often study the measured quantities component by component, without taking into account their three-dimensional nature and the geometric properties of their trajectory. A significant part of the information is thus ignored. This article details a method dedicated to the analysis and processing of three-component quantities, capable of highlighting the special geometric features of such data and providing complementary information for condition monitoring. The proposed method is applied to two experimental cases: bearing fault monitoring in rotating machines, and voltage dips monitoring in three-phase power networks. In this two cases, the obtained results are promising and show that the estimated geometric indicators lead to complementary information that can be useful for condition monitoring.

1. Introduction

Safety and economical constraints force industries to continuously improve their maintenance strategies. When possible, predictive or condition-based maintenance is used as it helps reducing repair time and cost, improve safety, and avoid economic losses. Very often, condition monitoring techniques rely on the characterization of inherently three-component physical quantities, which are frequently encountered in technological processes. A first example is the monitoring of three-phase electrical systems, based on three-phase electrical measurements like voltages and currents. Another common example is the monitoring of mechanical systems, based on three-axis vibration or three-dimensional displacement measurements. In order to obtain efficient fault indicators, such three-component signals are usually analyzed with the usual marginal and/or joint analysis tools in the time domain (correlation functions and/or correlation matrices) as well as in the frequency domain (spectra and/or spectral matrices) [1]. However, three-component signals also contain another type of information which is completely different in nature: their geometric properties. When a three-component signal is represented in three-dimensional Euclidean space, it follows a particular trajectory. The geometric properties of this trajectory may contain information concerning the state of the monitored system from which the signal was acquired. This approach has already been successfully proposed in the field of system monitoring with two-component signals [2] by using complex-valued

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signal processing tools [3]. It is for example the case for orbit shape analysis used to detect faults in rotating machines [4–6], and for voltage dips detection and classification in power networks [7]. However, and as previously mentioned, usual condition monitoring methods do not take into account the three-dimensional geometric characteristics of the trajectory of the measured three-component quantities. As a consequence, a significant part of the diagnostic information is ignored.

This research work aims to fill this gap by developing a method to estimate the geometric properties of three-component signals which takes into account all three components at the same time. The proposed method relies on basic concepts of differential geometry of space curves such as the Frenet-Serret frame and formulas, curvature and torsion, and leads to local geometric descriptors of the three-dimensional curves followed by three-component signals. The method takes as its input a three-component signal, i.e. a time series where three data points are available at each time t. These data are then considered as Cartesian coordinates defining the position of the measured signal at time t in a three-dimensional Euclidean space. However, raw signals measured in real-life systems tend to be complicated and thus lead to trajectories with complicated geometric properties. To simplify matters, as in spectral analysis, the signal is simplified by analyzing only one frequency component. This sinusoidal signal, composed of three sinusoids of the same frequency, follows a trajectory in three-dimensional space which is elliptical in shape, and the geometric properties of the corresponding trajectory can be analyzed more easily. This is what the proposed method is for: to estimate the geometric properties of the trajectory of a three-component sinusoidal signal in three-dimensional space. The estimated geometric properties can then be used to elaborate stand-alone or complementary fault indicators for condition monitoring purposes.

In order to validate this approach, the proposed method is applied to two different experimental cases: voltage dips monitoring in three-phase power networks, and bearing faults monitoring in rotating machines. In this two cases, the results obtained are promising and show that the estimated geometric indicators lead to complementary information that can be useful for condition monitoring purpose.

The previous ideas are detailed in this article, which is organized as follows. The theoretical foundations of the proposed method are presented in Section 2, which includes the basic differential geometry tools used, the definition of the three-component signal of interest, as well as the geometric properties to be estimated. The structure of the algorithm developed to estimate these geometric properties is then described in Section 3, along with the details of its estimation performance with respect to various parameters. The experimental results obtained by this algorithm in the context of the two application examples of voltage dips monitoring in three-phase power networks and bearing faults monitoring in rotating machines are given in Sections 4 and 5 respectively. Finally, the article ends with concluding remarks including a summary and suggestions of possible future work.

2. Geometric properties of three-component sinusoidal signals

2.1. Differential geometry of space curves

A natural way of defining a curve is through differentiable functions. Let I be an open interval in the real line $\mathbb{R}$ and $\mathbf{r}$ be a function from I to $\mathbb{R}^3$ as defined in Eq. (1) where $\mathbb{R}^3$ denotes the set of triples of real numbers and $r_1,\ r_2$ and r_3 are differentiable functions of t.

$$\begin{aligned} &\mathbf{r} : I \to \mathbb{R}^3 \\ &t \mapsto (r_1(t), r_2(t), r_3(t)) \end{aligned} \tag{1}$$

$\mathbf{r}$ is called a parametrized differentiable curve and the variable t is the parameter of the curve [8,9]. A curve $\mathbf{r}$ maps each t in I into a point $\mathbf{r}(t) = (r_1(t), r_2(t), r_3(t))$ in $\mathbb{R}^3$ in such a way that the functions r_1, r_2 and r_3 are differentiable. In other words, at each t in some open interval I, $\mathbf{r}$ is located at the point $\mathbf{r}(t) = (r_1(t), r_2(t), r_3(t))$ in $\mathbb{R}^3$, and the corresponding curve can be pictured as a trip taken by a moving point $\mathbf{r}$ in $\mathbb{R}^3$.

The Frenet-Serret frame is the most natural choice to study the local geometric properties of a curve. It can be interpreted as a moving reference frame that provides a local coordinate system at each point of the curve, facilitating the definition of geometric properties of the curve in the neighborhood of each point [10]. The Frenet-Serret frame is composed of three orthogonal unit vectors $\mathbf{T}(t),\ \mathbf{N}(t)$, and $\mathbf{B}(t)$, respectively called the tangent, normal and binormal vector. Their definition is given by the three following equations [9,11]:

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} \tag{2}$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|} = \frac{\mathbf{r}'(t) \times (\mathbf{r}''(t) \times \mathbf{r}'(t))}{\|\mathbf{r}'(t)\| \|\mathbf{r}''(t) \times \mathbf{r}'(t)\|} \tag{3}$$

$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t) = \frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|} \tag{4}$$

where $\mathbf{r}'(t) = \frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t}$ is the derivative of $\mathbf{r}(t),\ \times$ denotes the cross-product between two vectors and $\|\mathbf{r}(t)\|$ is the norm of $\mathbf{r}(t)$ defined by: