Mechanical Systems and Signal Processing 99 (2018) 14–29

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Wavelet-bounded empirical mode decomposition for measured time series analysis

Keegan J. Moore [a,*], Mehmet Kurt [b], Melih Eriten [c], D. Michael McFarland [d], Lawrence A. Bergman [d], Alexander F. Vakakis [a]

[a] Department of Mechanical Science and Engineering, University of Illinois, Urbana, IL 61801, United States
[b] Department of Bioengineering, Stanford University, Stanford, CA 94305, United States
[c] Department of Mechanical Engineering, University of Wisconsin at Madison, Madison, WI 53706, United States
[d] Department of Aerospace Engineering, University of Illinois, Urbana, IL 61801, United States

ARTICLE INFO

Article history:
Received 18 December 2016
Received in revised form 22 May 2017
Accepted 4 June 2017

Keywords:
Empirical mode decomposition
Wavelet transform
Nonlinear analysis

ABSTRACT

Empirical mode decomposition (EMD) is a powerful technique for separating the transient responses of nonlinear and nonstationary systems into finite sets of nearly orthogonal components, called intrinsic mode functions (IMFs), which represent the dynamics on different characteristic time scales. However, a deficiency of EMD is the mixing of two or more components in a single IMF, which can drastically affect the physical meaning of the empirical decomposition results. In this paper, we present a new approached based on EMD, designated as wavelet-bounded empirical mode decomposition (WBEMD), which is a closed-loop, optimization-based solution to the problem of mode mixing. The optimization routine relies on maximizing the isolation of an IMF around a characteristic frequency. This isolation is measured by fitting a bounding function around the IMF in the frequency domain and computing the area under this function. It follows that a large (small) area corresponds to a poorly (well) separated IMF. An optimization routine is developed based on this result with the objective of minimizing the bounding-function area and with the masking signal parameters serving as free parameters, such that a well-separated IMF is extracted. As examples of application of WBEMD we apply the proposed method, first to a stationary, two-component signal, and then to the numerically simulated response of a cantilever beam with an essentially nonlinear end attachment. We find that WBEMD vastly improves upon EMD and that the extracted sets of IMFs provide insight into the underlying physics of the response of each system.

1. Introduction

The empirical mode decomposition (EMD) proposed by Huang et al. [1] is a powerful method for decomposing oscillatory signals into a finite basis of nearly orthogonal, monochromatic intrinsic mode functions (IMFs) each possessing a characteristic time scale. EMD is highly suitable for application to nonlinear and nonstationary processes where linear decomposition methods fail to capture the complex nonlinear dynamics. EMD decomposes an oscillatory signal, $u(t)$, with a sifting algorithm consisting of the following steps:

* Corresponding author.
E-mail address: kmoore14@illinois.edu (K.J. Moore).

http://dx.doi.org/10.1016/j.ymssp.2017.06.005
0888-3270/

Nomenclature

A	amplitude ratio between y_1 and y_2
$\hat{A}_i$	the discrete set of instantaneous amplitudes for the ith set of IMFs
B	area under bounding function
$\tilde{C}_i$	maximum wavelet transform of an IMF c_i
$\mathrm{HT}\{\cdot\}$	Hilbert transform
$\mathrm{MWT}\{\cdot\}$	maximum wavelet transform
R	average of minima and maxima envelopes
$\mathrm{WT}\{\cdot\}$	wavelet transform
Z	wavelet transform of z
$\tilde{Z}$	maximum wavelet transform of z
b	bounding function
c_i^m	the ith IMF extracted at the mth measurement location
$e_{\min}(e_{\max})$	the envelope created by spline interpolating minima (maxima)
f_1	frequency of y_2
k	frequency ratio between ω_i and ω_{i+1}
s	masking signal
u	an oscillatory signal in time (measured or simulated)
y	two-component signal composed of y_1 and y_2
z_i	the difference between u and the sum of all IMFs 1 through i
$\Phi_{r:p}$	phase variable defined for a ratio of $r:p$
Ω	upper frequency limit for computing area under bounding function
α	parameter that adjusts the masking signal amplitude
β	parameter that adjusts the masking signal frequency
γ	minimum amplitude of a trial IMF at the characteristic frequency
δ	maximum amplitude of a trial IMF away from the characteristic frequency
ε	parameter that controls the minimum value of the bounding function
η	parameter that adjusts the bounding function peak amplitude
ϕ	parameter that adjusts the width of the bounding function
$\hat{\theta}_i(\hat{\omega}_i)$	instantaneous phase (frequency) of an IMF c_i
$\psi_{\omega,t}$	mother wavelet function
ω_i	the ith characteristic frequency

(i) Determine all extrema of $u(t)$.
(ii) Compute two envelopes, $e_{\min}(t)$ and $e_{\max}(t)$, by spline interpolating the minima and maxima of the signal.
(iii) Compute the average curve between the two envelopes, $R(t) = (e_{\max}(t) + e_{\min}(t))/2$.
(iv) Extract the remainder signal, $c_1(t) = u(t) - R(t)$.

Apply steps (i) through (iv) iteratively to $c_1(t)$ until the maximum value of $R(t)$ is less than a prescribed tolerance, τ.

Upon satisfying (iv), $c_1(t)$ is regarded as the first IMF of $u(t)$ and possesses the highest characteristic frequency. A second IMF can be extracted by applying the algorithm to the difference $z_1(t) = u(t) - c_1(t)$. By applying the algorithm recursively, $x(t)$ can be sequentially decomposed into nearly orthogonal IMFs that satisfy

$$u(t) = \sum_{i=1}^{N} c_i(t) + R_{N+1}(t), \max(R_{N+1}(t)) < \tau. \tag{1}$$

The objective of EMD, from an applications viewpoint, is to extract IMFs that are physically and mathematically representative of the original time series. Although the mathematical and physical significance of IMFs have been studied in detail [2–4], applying EMD often results in more IMFs than the number of characteristic time scales actually present (i.e., the method yields spurious, non-physically meaningful IMFs), and care must be taken to select only the physically meaningful IMFs from the extracted ones [1,5,6]. Other tools, such as the wavelet transform (WT) [7], provide great insight into the characteristic frequencies and their temporal behavior. These tools should be applied to identify the characteristic frequencies before applying EMD.

In addition to spurious IMFs, EMD suffers from a lack of a theoretical foundation; orthogonality between IMFs; uniqueness in the decomposition; and mode mixing where EMD results in multi-frequency IMFs. Huang et al. [5] first described