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Model selection and parameter estimation in structural dynamics using approximate Bayesian computation

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A B S T R A C T

This paper will introduce the use of the approximate Bayesian computation (ABC) algorithm for model selection and parameter estimation in structural dynamics. ABC is a likelihood-free method typically used when the likelihood function is either intractable or cannot be approached in a closed form. To circumvent the evaluation of the likelihood function, simulation from a forward model is at the core of the ABC algorithm. The algorithm offers the possibility to use different metrics and summary statistics representative of the data to carry out Bayesian inference. The efficacy of the algorithm in structural dynamics is demonstrated through three different illustrative examples of nonlinear system identification: cubic and cubic-quintic models, the Bouc-Wen model and the Duffing oscillator. The obtained results suggest that ABC is a promising alternative to deal with model selection and parameter estimation issues, specifically for systems with complex behaviours.

1. Introduction

In many areas of engineering and science, researchers or engineers are dealing with model selection and comparison issues, in particular when several competing models are consistent with the selection criteria and could potentially explain the data reasonably well. In most cases, selecting the most likely model among a set of competing models may be quite challenging, often requiring a deep understanding of the physics involved. Several methods have been proposed in the literature, and arguably the most popular currently is the Bayesian approach. During the last two decades, the Bayesian approach has been successfully implemented in many areas to deal with model selection and parameter estimation issues. Compared with other statistical methods, Bayesian theory provides a comprehensive and coherent framework, and a generally applicable way to make inference about models from data. The reader can refer to the following references [1–6] and the references therein, where many varied examples illustrating the use of the Bayesian method are investigated. In the Bayesian paradigm, the best model is the one that satisfies the parsimony principle, which means the right balance between complexity of the model and goodness-of-fit. Given a number of potential models, and one or more data sets, model selection should identify the model structure and the set of parameters that may explain the data best, while simultaneously penalising overly-complex models. Different methods have been proposed in the literature for model selection based on the Bayes theory; the most popular is reversible-jump Markov chain Monte Carlo (RJ-MCMC) [7]. However, the implementation of the RJ-MCMC algorithm is quite challenging. This is because when one deals with a large number of models with different

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dimensionalities, the algorithm needs to define a so-called 'dimension-matching' mapping law which requires additional computation. The reader is referred to [7] for details. Bayes factors [8] have been considered for a long time as the standard tools for performing Bayesian model comparison; however, these provide only a relative comparison of competing models, not the absolute values of their posterior probabilities.

Sandhu et al. [5] have proposed the use of the Metropolis-Hastings (MH) MCMC simulation and nonlinear filtering. Particle filters as the sequential importance sampling/resampling (SIS/SIR) [9] could be used to make model selection as shown in [10]. More traditional statistical methods such as the Akaike Information Criterion (IC), the Bayesian IC or the deviance IC have been extensively used and investigated in the literature also [11–14]. Essentially, the evaluation of those metrics is based on the maximum likelihood estimate and a penalty term to penalise complex models (complexity is measured usually by the number of parameters in the model). In those methods, the marginal likelihood estimation is undertaken for each model separately, and then these results are used to compute the plausibility of each model. This may be a problem, typically when one deals with a large number of competing models composed of a large number of parameters. Moreover, in the statistical methods based on the ICs, the likelihood is supposed to be very peaked, however, in problems with different types of nonlinearities, the density may be non-Gaussian (e.g., bimodal, multimodal or heavily skewed). In such cases, the ICs cannot be used to compare the candidate models, and this limits their widespread use. Another alternative to deal with model selection and parameter estimation is to use the nested sampling (NS) method proposed by Skilling [15,16]. The algorithm works by transforming the multidimensional parameter space integral into a one dimensional integral where classical numerical approximation techniques to estimate the area under a function can be applied. The algorithm has been successfully applied in various research areas [17,18].

The diversity of the methodologies proposed in the literature reflects the complexity of the model selection task; moreover, it shows that there is no universal method that can be used in any circumstances. The choice of the suitable method depends mainly on the available data to conduct Bayesian inference. In this paper, the use of the approximate Bayesian computation (ABC) algorithm is introduced as a promising alternative to deal with model selection and parameter estimation. Compared with the methods above, the ABC is more straightforward and general in the sense that there is no need to evaluate any extra criterion to discriminate between candidate models, and the inference can be performed through any suitable metric to assess the similarity between the observed and simulated data, circumventing the problem of an intractable likelihood function and the Gaussianity assumption which cannot not always be guaranteed. Moreover, in structural dynamics with complex nonlinearity types, it is often the case that the hypothesis of Gaussianity is not guaranteed. Another major advantage offered by the ABC algorithm is its independence of the dimensionality of the competing models; ABC jumps between the different model spaces without the need of any mapping function to be defined, which is a major benefit in dealing with larger numbers of models. In practice, the ABC algorithm compares the competing models simultaneously, and eliminates progressively the least likely models, to converge to the most plausible one(s). The widespread use of ABC in several fields, and its efficiency to deal with model selection and parameter estimation, simultaneously motivated the current authors to investigate more the capability of the ABC to infer complex nonlinear systems in structural dynamics. The algorithm shows some attractive properties, including its flexibility to use different kinds of metrics to make system inference and its ability to explore both model and parameter spaces efficiently. The flexibility offered by the ABC algorithm is of paramount importance, as in some circumstances, the likelihood function cannot be analytically formulated or even be approached using approximate methods. Therefore, ABC by its flexibility makes inference possible for many challenging problems.

During the last decade, the ABC algorithm has been applied in many areas for both levels of inference (parameter and model): genetics [19], biology [20,21] and psychology [22]. The rapid developments and continuous improvements of the ABC algorithm attracted many other areas, and recently it has been introduced in structural dynamics by the authors for model selection [23] and parameter estimation in [24]. In [24], the authors show that the combination of the ABC principle with the subset simulation concept [25], introduced to estimate rare events, decreases the computational time and provides the same precision as other variants of the ABC algorithm proposed in the literature such as ABC sequential Monte Carlo (SMC) and ABC-MCMC [26]. In the present work, a more extensive application of ABC-SMC as an efficient tool for model selection and parameter estimation in structural dynamics is presented. ABC appears to be a promising alternative for practitioners in structural dynamics to overcome the inference problem of systems with complex behaviours which may undergo bifurcations and/or chaos.

Furthermore, ABC offers the possibility to manage larger datasets and a higher number of competing models with different dimensionalities, circumventing the limitation of RJ-MCMC. Besides the major advantages mentioned so far, the simplicity of the ABC method and its capability of extending the Bayesian framework to any computer simulation has exponentially increased its popularity. It is worth mentioning that this algorithm already takes into account the parsimony requirement because complex models with larger number of parameters will generate wider posterior distributions. As a result, models with more parameters will be more times below the ABC tolerance threshold, thus promoting simpler models. This property will be investigated in the illustrative examples presented in this work, by considering several models with different degrees of complexity and analysing the behaviour of the algorithm through the inference process.

The paper starts out with an introduction to the ABC algorithm and the selection of the different hyperparameters required for its implementation. Then, in Section 3, the application of the ABC algorithm is illustrated and investigated through three illustrative examples using simulated data and forms the core of the paper. Finally, the paper is closed with some conclusions about the strengths of the ABC method and future work.