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Modelling vertical human walking forces using self-sustained oscillator

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ABSTRACT

This paper proposes a model of a self-sustained oscillator which can generate reliably the vertical contact force between the feet of a healthy pedestrian and the supporting flat rigid surface. The model is motivated by the self-sustained nature of the walking process, i.e. a pedestrian generates the required inner energy to sustain its repetitive body motion. The derived model is a fusion of the well-known Rayleigh, Van der Pol and Duffing oscillators. Some additional nonlinear terms are added to produce both the odd and even harmonics observed in the experimentally measured force data. The model parameters were derived from force records due to twelve pedestrians walking on an instrumented treadmill at ten speeds using a linear least square technique. The stability analysis was performed using the energy balance method and perturbation method. The results obtained from the model show a good agreement with the experimental results.

1. Introduction

Civil engineering structures such as footbridges, floors, grandstands and staircases are commonly subjected to dynamic loads due to normal human activities such as walking, running and jumping [1,2]. Architectural trends towards slender structural design and using modern high-strength construction materials often result in natural frequencies below 5 Hz that are within the range of common human actions, thus creating serious vibration serviceability problems [1–4]. Excessive lateral vibrations induced by pedestrians walking observed on the London Millennium bridge [5], the Paris Solferino bridge [6] and the Toda Park bridge [7] are just a few notable examples that triggered research on human-structure interaction (HSI) [1,8–10] and lateral walking loading [3,11]. While the lateral walking excitation takes a significant role only in design of footbridges, the corresponding vertical forces are also needed for design of all other types of structures that are commonly occupied by humans. This is because walking is the most common human activity and the dynamic amplitude of the vertical walking force is significantly higher [12]. Modelling of the vertical walking forces is also an important research subject in robotics [13,14] and biomechanics [15–18].

To date researchers used two different modelling approaches to describe the vertical walking loading. The most common is a mathematical description of the contact force between the feet and the ground [19–21], so called ground reaction force (GRF). The major disadvantage of this approach is that it neglects HSI, i.e. the effect of the body on the dynamic properties of

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the occupied structure [3,20]. Therefore, the second modelling approach describes a human body as a mechanical system, often composed of masses connected with springs and dampers. The simplest one is a linear oscillator having a single mass, spring and damping accompanied by a vertical GRF [22], followed by models of multiple lumped masses connected with linear springs and dampers [23–27]. More complex are biomechanically inspired inverted pendulum models [11,28–32] and multibody link segment models of the whole human body [33]. While the former do not replicate reliably the measured vertical force, the latter require long and computationally demanding simulations that are not suitable for structural design where time factor is often far too important.

Human walking is self-sustained, i.e. the body itself produces the inner energy to maintain its motion. Modelling moving humans as self-sustained oscillators has been already proved successful [34]. Most recently, Erlicher et al. [35,36] proposed a modified hybrid Van der Pol/Rayleigh single degree of freedom (SDoF) oscillator to describe the lateral pedestrian excitation of floor. The study presented in this paper builds on the knowledge from these past studies. Its objective is to model the vertical body motion during walking as an SDoF self-sustained oscillator which has (i) a stable limit cycle and (ii) can generate repetitive footfall forces observed in the actual vertical force records (see Section 4). Humans are not robots so the real walking is never perfectly repetitive but near-periodic. However, for the sake of simplicity, assuming periodicity of the vertical body movement during walking and the periodicity of the corresponding vertical forces is reasonable, bearing in mind the modelling scale taken in this study. The paper also describes walking on flat and rigid surfaces where structural vibrations are still not perceptible, thus do not alter the gait. Unlike the biomechanical models, the proposed mathematical model is not designed to reflect physiology of the human body. It describes a fictitious mechanical system able to replicate dominant Fourier harmonics observed in the experimentally measured walking force records.

The paper is divided into eight sections. This introductory section is followed by a brief overview of the experimental data collection in Section 2. Section 3 presents the results of the Fourier-based analysis of the recorded force signals. Based on the results, a self-sustained oscillator is derived in Section 4. The stability analysis of the modelling parameters is elaborated in Section 5, followed by identification of their values in Section 6. The results are summarized in Section 7. Finally, concluding remarks are presented in Section 8.

2. Experimental data acquisition

The vertical force signals due to 12 healthy individuals walking were recorded on an instrumented force measuring treadmill ADAL3D-F [37], which is permanently installed in the Light Structures Laboratory at the University of Sheffield. The number of test subjects was decided based on similar experiments designed for biomechanical studies of human gait, including clinical studies and sport biomechanics [3]. Later in Section 7 it will become evident that a dozen is just a few in the context of the present study, so a larger number would have increased statistical reliability of the research findings. A test protocol approved by the University Research Ethics Committee required each participant to complete a Physical Activity Readiness Questionnaire and pass a preliminary fitness test to check whether they were healthy and suited for the experiment. Mass, age, body weight and height were also recorded for each test subject (Table 1). All participants wore comfortable flat sole shoes. Before data collection, they did approximately ten minutes long warming up that included walking on the treadmill at random speeds, so that differences between treadmill and over ground walking vanished even for inexperienced test subjects [38].

During ten tests the walking forces from each participant were recorded at ten constant treadmill speeds in the range 2–6.5 km/h with the increment 0.5 km/h. The speeds close to the range borders are unnaturally slow and fast to study extreme situations. Pacing rate was not prompted by any stimuli such as a metronome, and it was determined only from subsequent analysis of the generated force signals (Section 3). The sampling rate f_s was 200 Hz. Each force record contains at least 64 successive steps. A typical force record is shown in Fig. 1.

3. Preliminary analysis of the experimental force signals

This section provides a solid foundation for the development and parameter identification of the self-sustained oscillator presented in Sections 4 and 5. Section 3.1 studies the amplitude and phase Fourier spectra of the force records measured in Section 2. The results are used in Section 3.2 to derive the vertical kinematics of the walking model and to briefly study displacement-velocity-force relationships for individual pedestrians.

Table 1
General data from the test subjects. The participants are arranged according to their body mass.

	A	B	C	D	E	F	G	H	I	J	K	L
Age	57	30	40	22	28	33	46	21	32	41	33	44
Gender	M	F	F	M	M	F	M	M	M	M	M	M
Height [cm]	163	166	165	188	178	171	168	188	169	173	180	183
Body mass [kg]	50.97	55.56	56.98	60.04	62.69	63.1	64.02	64.53	80.02	81.86	82.77	85.83