Mechanical Systems and Signal Processing 99 (2018) 364–383

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Localisation of local nonlinearities in structural dynamics using spatially incomplete measured data

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ARTICLE INFO

Article history:
Received 23 July 2016
Received in revised form 16 June 2017
Accepted 19 June 2017

Keywords:
Nonlinear model updating
Spatially incomplete measurement
Localisation of nonlinear elements

ABSTRACT

This paper presents a procedure to localise nonlinear elements using spatially incomplete measured frequency response data from the structural vibration test. The method does not require measurements of all the responses associated with nonlinear elements and the information about the types of nonlinear elements. In this procedure, the Craig-Bampton reduction method is employed to reduce the dynamic equation onto the measured region and to project the nonlinear forces onto the measured degrees of freedom (DOFs), which are then called reduced nonlinear forces (RNFs). It is shown that the reduced nonlinear forces are the sum of the measured nonlinear forces and the projections of the unmeasured nonlinear forces through the transpose of linear constraint modes. Therefore, by analysing and comparing the magnitude of the reduced nonlinear forces obtained from experiments with the linear constraint modes, we can localise the nonlinear elements without directly measuring their responses. Numerical simulations of a discrete system with two nonlinearities and experimental data from a clamped beam with a nonlinear connection are used to validate the localisation procedure.

1. Introduction

Nonlinearity exists in a wide range of engineering structures and plays a key role in complicated structural behaviours such as multi-value responses and input-dependent frequency response functions. In contrast to the well-established modal testing schemes [1] or updating procedures [2] for linear structures, there is no universal or standard approach to experimental test of nonlinear structures, despite considerable research efforts.

The localisation of nonlinear elements, which is the subject of this paper, is an important step in the identification framework of multiple-degree-of-freedom (MDOF) nonlinear structures [3–6]. It can be used to detect unexpected faults, such as nonlinear cracks or failures, or unexpected nonlinear boundary conditions that were originally assumed to be linear. Using the location of nonlinear elements to separate the underlying linear and nonlinear parts of a MDOF structure has already proved to be a useful and robust way to formulate a well-conditioned identification procedure, especially for practical engineering structures with many DOFs. The locations of nonlinear elements can also reveal clear physical insights into the system and provide the necessary information to add additional nonlinear elements when improving the Finite Element (FE)

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http://dx.doi.org/10.1016/j.ymssp.2017.06.021

model. Finally, the localisation should confirm engineering experience of real structures and give confidence in the results, i.e. potential nonlinear locations should correspond to the existence of joints, bearings, contacts, nonlinear materials, etc.

Input-dependent frequency response functions, and the lack of conventional modal decoupling and modal superposition techniques, provide significant challenges and raise several questions about the localisation process of nonlinear elements. The first question is whether the types of the nonlinear elements should be known before the localisation begins. Early studies [7–9] assumed that the types of nonlinearity should be pre-known or pre-identified. Candidate nonlinear forces (with some unknown variables) were applied to each DOF of the structure and the unknown variables were identified from the measured nonlinear responses; the DOFs with non-zero variables determine the locations of the nonlinear elements. Kerschen et al. [3] recommended the localisation process as a second step after the detection of the nonlinearities, and thus no priori information of the nonlinearities should be assumed. Recently, Ewins et al. [4] placed 'characterisation of nonlinearity' ahead of 'location of nonlinearity' in their 'modal+' testing procedure, which naturally uses the type of nonlinearity as a priori information during the localisation process. In practice, it seems unlikely that the type of nonlinearity would be known, but the location would be known. One possibility is to build a library of every possible type of nonlinearity and use a brute-force method to explore every combination; this is very time consuming and risks numerical instability for structures with many DOFs.

Another question is whether all of the responses associated with the nonlinear elements have to be measured during the localisation process, which is equivalent to limiting the possible locations of the nonlinear elements to within the measured region. By using force identification methods [10,11], the DOFs associated with the nonlinear elements do not have to be measured. However, the measured frequency response function (FRF) matrix must be inverted to extract the nonlinear forces, and this inversion is sensitive to the measurement noise and modelling errors. For linear structures we can use the modal superposition techniques or the nonlinear perturbation of the linear receptance to avoid the inversion [12]; however, this is not possible for nonlinear structures since superposition is no longer valid. A compromise is to assume that all of the DOFs associated with the nonlinear elements are included in the measured region [13–15], as shown in Fig. 1(a). As such, the dynamic equation can be partitioned into the measured and unmeasured regions. Since no nonlinear forces are assumed in the unmeasured region, the unmeasured responses can be estimated by the expansion of the measured responses [13–15]. This restriction is a severe limitation for the localisation of nonlinear elements [3,16], since for practical applications the DOFs directly associated with the nonlinear element are unlikely to be fully measured when their locations are unknown.

The objective of this paper is to remove the restriction in the localisation procedure that requires full measurements of the DOFs associated with the nonlinearities, without using a priori information about the types of nonlinearities, as shown in Fig. 1(b). The localisation procedure starts with an initial linear FE model constructed for the underlying linear structure and updates it by the well-developed linear model updating technique [2] using data measured under a low amplitude excitation. Next, the reduced nonlinear forces are extracted by correlating the updated linear FE model to the data from the high amplitude stepped-sine excitation, and these forces are then compared with the linear constraint modes to determine the candidate DOFs included in the suspect region (possible location region for nonlinear elements), prior to finally localising the measured or unmeasured nonlinear elements and verifying the localisation results.

The rest of this paper is organised as follows. In Section 2, the proposed localisation procedure is introduced in detail. In Section 3, the localisation procedure is demonstrated and discussed using a twenty-DOF numerical example with two nonlinearities. An application of this procedure to localise a nonlinear mechanism of a clamped beam using experimental data is presented in Section 4. Conclusions are drawn in Section 5.

2. The localisation procedure

2.1. Formulation

Consider the general form of the dynamic equation of a MDOF discrete system with multiple nonlinearities as

$$\mathbf{M}\ddot{\boldsymbol{x}} + \mathbf{C}\dot{\boldsymbol{x}} + \mathbf{K}\boldsymbol{x} + \boldsymbol{f}^{*}(\boldsymbol{x}, \dot{\boldsymbol{x}}) = \boldsymbol{p}(t), \quad (1)$$

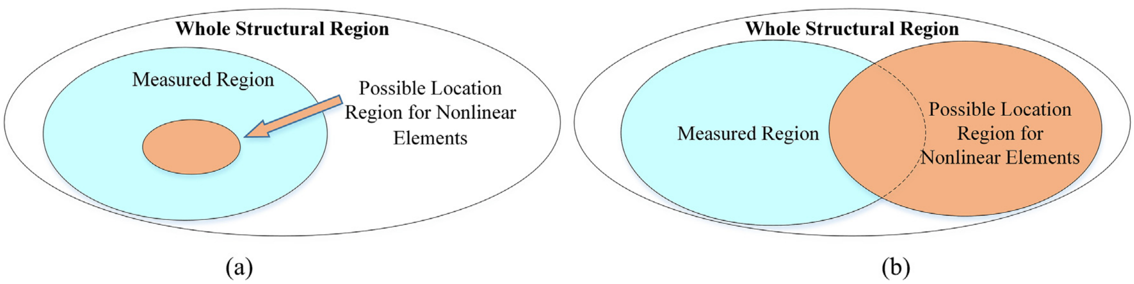

Fig. 1. The region assumption of (a) the previous localisation procedure [13–15] and (b) the proposed localisation procedure.