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Nonlinear characterization of a Rossler system under periodic closed-loop control via time-frequency and bispectral analysis

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A B S T R A C T

This study has two primary objectives; they are to investigate the nonlinear interactions (or quadratic phase-coupling) in a chaotic Rossler system under periodic closed-loop control via wavelet bispectral analysis; and to further identify the component mechanisms of synchronization. It is observed that a fixed-gain, fixed-frequency controller produces quadratic phase-coupling and decoupling along lines of constant frequency and that are perpendicular to the diagonal of the bicoherence matrix. Further, it was also observed that for synchronization to occur, *both* frequency entrainment and quadratic phase-coupling *must* be present. It was found that forcing the Rossler system with a constant frequency did not reduce the amplitude of the resulting period-1 orbit at sufficiently high gains. For the controller with a fixed gain and time-varying error signal, it was found that the time varying forcing frequency (adjusted by an extremum seeking feedback loop) linearizes the Rossler system and in doing so, suppresses the phase coherence completely. The time-varying forcing frequency removes the conditions for frequency entrainment by providing broadband attenuation; the result is suppression without synchronization.

1. Introduction

Power spectral analysis is sufficient for the analysis of linear systems, but they cannot provide information about the nonlinear interaction between Fourier modes, nor can they resolve the changes in Fourier components in time. In general, the bicoherence, which is the normalized bispectrum a measure of the amount of phase coupling that occurs in a signal or between two signals. Phase coupling is said to occur when two component frequencies are simultaneously present in the signal (s) along with their sum (or difference) frequencies and the phase of these component frequencies remains constant. There are two types of bicoherence analysis, the first is Fourier based and the other is wavelet based; the wavelet based option will be used in this research. A formal definition of wavelet bicoherence is provided in Section 3 of this work. Bispectral analysis is applied to a wide variety of nonlinear systems. These systems include mathematical nonlinear systems with quadratic and cubic nonlinearities, mechanical systems, aeromechanical systems and fluid mechanics. The following brief literature review will highlight some of these examples.

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Pezeshki et al. [1] performed a Fourier based auto-bispectral analysis on an unforced Rossler system with parameters *a* and *b* fixed at 0.2 and the parameter *c* was set at three discrete values of 2.6, 3.5 and 4.6. It was found that the nonlinear modal interactions were completely characterized by the bispectral analysis; this is due to the fact that the Rossler system equations contain a quadratic nonlinearity. For the period-1 motion ($c = 2.6$), it was found that spectrum contained a peak at 0.17 Hz and higher harmonics (typical for the Rossler system when the parameters are equal to 0.2). The corresponding bicoherence map showed self-coupling at 0.17 Hz and 0.34 Hz. For the period-2 motion ($c = 3.5$), it was observed that there is coupling between motions at the fundamental frequency and the period double frequency and its harmonics. It is the energy transfer between the fundamental and the period double frequency of 0.34 Hz that are responsible for the amplification of the frequency peaks seen in the spectrum. When the system is in the chaotic regime ($c = 4.6$), the bicoherence map actually fills with more contours and has similar features as the period doubling case. The analysis for this paper will be performed for $c = 5.7$. Classically, when the parameter *c* is set equal to 5.7, the Rossler system is simultaneously chaotic and phase coherent, making it more challenging to control.

In [1] a magnetically buckled beam was also studied. This mechanical system can be modeled as a Duffing's oscillator. A Duffing's oscillator is typically a second order system with a negative linear stiffness and a cubic nonlinearity. The Duffing oscillator is driven by a periodic forcing term in which the forcing frequency is fixed and the forcing amplitude is set equal to three values; each corresponding to a period-1, period-2 and chaotic regimes. The evolution of the bicoherence for the first two forcing amplitudes is similar to that of the Rossler system in that the quadratic phase-coupling appears to form along lines of constant frequency that match those found in the spectra. As the forcing amplitude increases, the 'bands' of bicoherence starts to increase in width; in the context of a forced system, quadratic phase coupling leads to the transfer of energy from the high frequency forcing to the lowest natural nonlinear frequency. This low frequency energy is then redistributed to many frequencies which eventually produce frequency spreading. The corresponding effect in the bicoherence matrix is the widening of the bands. Some nonlinear systems are self-excited, and as such the fundamental frequency will shed its energy to subharmonic, inter and superharmonic frequencies. When the Duffing oscillator is chaotic, the bicoherence map becomes diffuse and diminishes. This map feature is caused by the fact that nonlinearity is cubic; the influence of the cubic nonlinearity becomes stronger as the forcing amplitude is increased. Furthermore, the analysis method is of order two thus when the cubic nonlinearity becomes pronounced, it is possible for the magnitude of the bicoherence to have a value of zero. Auto and cross bispectral analyses of a two degree-of-freedom system with quadratic nonlinearities having a two-to-one internal (autoparametric) resonance are presented [2]. It was found that the bispectral analysis method characterized the nonlinear modal interaction both inside and out of the chaotic regime. Specifically, for the periodic orbits 1, 2, 8, 16 and the chaotic regime, all possessed strong bicoherence that originated primarily from the fundamental frequency of the system. There was negligible bicoherence observed between the amplitude modulations when in the chaotic regime. This bicoherence map feature may be typical of non-autonomous driven oscillators. It was also suggested that when there is little energy transfer between amplitude modulations, the modes are decoupled; if the frequency components are decoupled, then the system cannot exhibit a chaotic response. In Balachandran and Khan [3], a forced nonlinear oscillator and a set of coupled forced nonlinear oscillators with quadratic nonlinearities are studied. The main contribution of this work was that the method of scales was used to aid in the development of analytical approximations for the bispectra. The analytical relationships for the bispectra showed a dependence on phase and magnitude of the bispectra on various system parameters. Pasquali et al. [4] demonstrated both numerically and experimentally that higher order spectral analysis tools are able to identify structural nonlinearities in physics-based mechanical plate models (elastic and laminated) and experimental data.

Silva and Hajj [5] examined the experimental results from the High Speed Civil Transport (HSCT) Flexible Semispan Model. Using Fourier based bispectral analyses (auto and cross-bispectral analysis), they were able to study the nonlinear interactions between the structural deflections of the semispan model and the surface pressure fluctuations. This aeroelastic study provided a further insight into the nonlinear flutter mechanism. Chabalko presents an extensive application of both classical signal processing techniques and higher order spectral methods (both Fourier and Wavelet based) to study the hard flutter response of the HSCT and the F-16 Limit Cycle Oscillation [6]. Jamsek and his colleagues [13] extended the bispectral analysis technique to encompass a non-stationary version of the calculation called time-phase bispectral analysis. Time-phase couplings can be observed by additionally calculating the adapted bispectrum from which the time varying bi-phase and bi-amplitude can be obtained. The bi-phase and bi-amplitude provide a means by which one can distinguish the type of coupling i.e. linear or quadratic in a single measurement. Jamsek et al. then proceeded to test out the method on Poincare oscillators. Their frequency and phase relationships were examined at different coupling strengths, both with and without noise and later coupled van der Pol oscillators and cardio-respiratory signals [14]. In both works it was found that linear, quadratic and parametric (frequency modulated) interactions can generally be distinguished from one another. All the calculations in these two works are Fourier based.

Typically, systems under closed loop control are examined using classical methods. Gilliat and Strganac have suggested that it is not possible to design a controller than can decouple a nonlinear aeroelastic system [10]. Suppose, now that wavelet bispectral analysis could be used to design control laws which target the quadratic phase coupling between the Fourier components that cause instability like flutter? It should be noted that a review of the literature shows that quadratic phase coupling is key to both chaotic and periodic responses, but not explicitly discussed is the phenomena of frequency entrainment. In general, frequency entrainment occurs when a periodic force is applied to a system whose free response is the self-excited type. Specifically, entrainment can occur if the ratio between the forcing frequency and the natural frequency is in the vicinity of an integer or a fraction.