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A new method of passive modifications for partial frequency assignment of general structures

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ABSTRACT

The assignment of a subset of natural frequencies to vibrating systems can be conveniently achieved by means of suitable structural modifications. It has been observed that such an approach usually leads to the undesired change of the unassigned natural frequencies, which is a phenomenon known as frequency spill-over. Such an issue has been dealt with in the literature only in simple specific cases.

In this paper, a new and general method is proposed that aims to assign a subset of natural frequencies with low spill-over. The optimal structural modifications are determined through a three-step procedure that considers both the prescribed eigenvalues and the feasibility constraints, assuring that the obtained solution is physically realizable. The proposed method is therefore applicable to very general vibrating systems, such as those obtained through the finite element method.

The numerical difficulties that may occur as a result of employing the method are also carefully addressed. Finally, the capabilities of the method are validated in three test-cases in which both lumped and distributed parameters are modified to obtain the desired eigenvalues.

1. Introduction

The assignment of natural frequencies (i.e. eigenvalues) is an important problem related to the design of vibrating systems. For example, in many applications it is wanted to keep the natural frequencies of the system far away from the dominant components of the harmonic excitation force, preventing resonance that can lead to structural failure. In contrast, in other cases, e.g. the design of resonators, it is wanted that a natural frequency of the system matches the single-harmonic excitation, in order to improve the performance of the machine and, at the same time, minimize the excitation effort.

It is well known that the natural frequencies are associated with the system eigenvalues, namely the solutions of the characteristic equation $\det(\lambda\mathbf{M} - \mathbf{K}) = 0$, given the mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$. Therefore, the appropriate mathematical framework for the assignment of natural frequencies is certainly constituted by the solution of inverse eigenvalue problems. Such problems concern the specification of the desired eigenvalues of the system and the determination of the suitable modifications of the system matrices that result in the required change of eigenvalues. Both passive [1–3] and active [4–6] approaches can be used to achieve such a goal, depending on the employed way of modifying the system matrices. In fact, the passive approaches consist in the structural modification, namely the adjustment of the physical parameters

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of the system, whereas the active approaches rely on feedback control through actuators and sensors. In the present paper, the assignment of natural frequencies through structural modification is addressed, which is a widely-employed approach that stands out for the low cost of implementation and the inherent stability of the resulting system.

The most useful case of natural frequency assignment is the one in which only a few eigenvalues are prescribed, because in practice often there is just a small number of dominant eigenvalues that characterize the dynamic behaviour. The modification of a subset of the natural frequencies, however, can lead to undesired change of the remaining ones, which is a phenomenon known as frequency spill-over. Design methods that prevent spill-over can avoid unexpected dynamics associated with the unassigned modes. In fact, the assignment of a subset of natural frequencies without spill-over, known in the literature as *partial eigenvalue assignment*, has become popular in very recent years. Active and passive approaches, however, tackle spill-over in a very different way. Indeed, in active control spill-over can make certain eigenvalues to become unstable, whereas passive modifications always maintain stability, as long as positive (semi-)definiteness of the system matrices are preserved. Nevertheless, even if instability is avoided, prevention of spill-over is desirable, to inhibit undesired resonances.

In active control, spill-over can be easily avoided, which can be manifested by the necessary and sufficient conditions available in the literature that ensure that the unassigned eigenvalues are kept unchanged. The earliest techniques for partial eigenvalue assignment rely on first order models, such as the projection and deflation technique [7] by Saad, or the parametric solution [8] by Datta and Sarkissian. Second order models are certainly more convenient since some properties of the system matrices, such as symmetry and sparsity, can be exploited. The discovery of orthogonality relations for the eigenvectors of the quadratic model by Datta et al. [5] stimulated the development of parametric solutions of the partial eigenvalue assignment problem [9,10]. The increasing complexity of the controlled vibrating systems determined the success of methods based on receptances, which can be experimentally measured, so they can be used even without complete knowledge of the system matrices. Indeed, partial eigenvalue assignment using receptances is achieved by Ghandchi Tehrani et al. in [11]. In [12], Singh et al. proposed a receptance-based method aimed at control of aerospace vehicles, which allowed for the placement of the poles due to aeroelasticity while ensuring the actuator poles remained invariant. Recently, a computationally efficient formulation was proposed by Bai and Wan in [13], by employing both receptances and system matrices, that led to the solution of a small-size linear system. A similar "hybrid" approach was proposed by Ram et al. to control a system with time-delay in [14], which was later extended to multi-input systems by Bai et al. in [15]. Partial eigenvalue assignment without spill-over for systems with time-delay has been addressed also by Singh et al. in [16]. Another relevant issue is robust control, which is studied in several papers about partial eigenvalue assignment [17–21]. It is worth to mention also the no spill-over condition for mass and stiffness modifications introduced by Zhang et al. in [22], which does not require the knowledge of the vibration modes to be retained.

The latter condition, however, cannot be trivially extended to passive approaches, because structural modifications must have a specified matrix structure to be feasible. Ensuring feasibility of the system matrices is always a challenge, even if all the eigenvalues are assigned and then spill-over is not an issue. Cai and Xu, for example, managed to preserve non-singularity or positive definiteness of the mass matrix [23]. Using a similar approach, Cai et al. in [24] dealt with the assignment of an incomplete set of eigenvalues, even though spill-over was not explicitly addressed. In [25], Mao and Dai propose a method for partial eigenvalue assignment for gyroscopic systems which preserved the mathematical structure but lost the physical realizability.

Available passive methods that ensure feasible solutions usually address only very simple specific problems. For example, Gürgöze and İnceoğlu in [26] considered the determination of the suitable spring to preserve the fundamental frequency of a beam with an added mass. Similarly, Mermertaş and Gürgöze [27] studied attachment of point masses and springs to plates keeping the fundamental frequency unchanged. In [28] Çakar examined the preservation of one natural frequency of a structure, neutralizing the shift of such a frequency caused by the addition of a number of masses through a suitable grounded spring. The partial eigenvalue assignment problem for general mass-spring structures, instead, was tackled by Ouyang and Zhang in [29], in which two methods were proposed: the first one concerned simply connected in-line mass-spring systems, while the second one dealt with multiple-connected mass-spring systems.

To overcome these limitations of known investigations reported in the open literature, the method proposed in this paper intends to extend the one in [29] to general vibrating linear systems with arbitrary matrix structures and modifiable parameters (e.g. distributed ones), such as those obtained with finite element modelling. The method of the cited paper exploits the mass-normalised stiffness matrix, which has the disadvantage that the physical parameters of the system cannot always be reconstructed, with few exceptions including the case in which the mass matrix is diagonal. In contrast, the proposed method deals with the mass matrix and the stiffness matrix separately, hence it is applicable to a wider class of vibrating systems.

Frequency assignment will be achieved in three steps: first of all, a system model that has the desired eigenvalues is sought, regardless of the constraints of physical feasibility. After that, an equivalent system is computed, which minimizes the deviation of the modifications from satisfying the feasibility constraints, by integration of a matrix differential equation. Finally, if it is necessary, the system is projected onto the feasibility constraints to obtain an optimal physically realizable structure.

The proposed method is validated in three different test-cases. First of all, a simple distributed-parameter system is employed to show the capabilities of the method in dealing with mass and stiffness matrices obtained from finite element modelling. Then the method is tested with a lumped parameter system, in order to provide a comparison with the state-of-