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A higher-order stress-based gradient-enhanced damage model based on isogeometric analysis

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Abstract

The micro-damage associated with diffuse fracture processes in quasi-brittle materials can be described by continuum damage mechanics. In order to overcome the mesh dependence of local damage formulations, non-local and gradient-enhanced approaches are often employed. In this manuscript, a higher-order stress-based gradient-enhanced formulation is proposed, which exploits the higher-order continuity of B-spline functions in isogeometric analysis (IGA). The proposed formulation does not require the decomposition of the fourth-order model into two second-order models. Two numerical examples are presented to demonstrate the performance of the formulation and to compare the obtained solutions with results from conventional gradient-enhanced damage formulation.

Keywords: Gradient enhanced damage; Isogeometric analysis; Higher-order functions; Stress-level dependent damage model

1. Introduction

Failure of structures made of quasi-brittle materials (such as cement based materials, masonry, biological or geological materials) is, in general, associated with the formation of cracks. Evidently, considerable efforts have been made to adequately represent the complex mechanisms underlying the opening and propagation of cracks in cohesive materials, leading to different computational strategies. The class of discrete crack models represent cracks as discrete C^{-1} discontinuities in the displacement field [1,2], incorporating the residual tractions in the cohesive zone by means of interface laws across the discontinuity [3–9]. The quasi-static crack growth can also be modeled by multiscale fracture models [10] such as the bridging domain method [11–13], the extended bridging domain method [14–16] and the adaptive multiscale method [17–20]. For modeling of cracks on an a priori macroscopic scale, the family of damage mechanics based models is often used to describe cohesive cracks as a damage zone which occupies a certain volume

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within the discretized structure (see, e.g. [21] for an overview of approaches). Damage mechanics [22] is a branch of continuum mechanics that describes failure by means of a continuous damage variable in conjunction with a loading function and a damage law. Non-local damage approaches [23,24] introduce an internal length scale guaranteeing the well-posedness of the boundary value problem. The implicit enhanced gradient formulation proposed by Peerlings et al. [25] can be considered as an effective non-local approach since the C^0 continuity requirement for the nonlocal damage parameters formulation enables a straightforward implementation into traditional finite element software. However, the inaccurate predictions of damage initiation and propagation of both integral and differential non-local damage formulations resulting from an insufficient description of interactions across cracks have been pointed out recently in [26–28]. In the case of strongly inhomogeneous strain fields, these formulations are unable to adequately describe the failure characteristics. Different failure modes arise and the failure load is predicted incorrectly. Linking the internal length with the stress level is an efficient remedy for this problem. A stress-based non-local damage model based on anisotropic weight functions was proposed by Giry et al. [29] to overcome the deficiencies of standard non-local approaches. In this formulation, the anisotropic weight function depends on the magnitude and the direction of the principal stress. It is employed to modify the internal length. As was demonstrated in [29], stress-based formulations are able to capture damage initiation and propagation more exactly in contrast to conventional non-local damage models. A stress-based gradient enhanced damage model using an anisotropic definition of the second order gradient operator was proposed by Simone and Bongers [30], defining separate coefficients related to derivatives with respect to the principal directions of the stress tensor. However, the effect of higher-order terms is neglected. In [31] the higher-order standard gradient enhanced damage formulations was developed taking the characteristic length as a constant material parameter. Different from [30,31], in this work, a higher-order gradient enhanced damage model is formulated in the framework of the stress based approach, where the characteristic length is a stress level dependent parameter.

To construct the higher-order gradient damage model, isogeometric finite elements are employed. The higher-order B-spline functions offer significant computational advantages in solving higher-order differential equations. Hughes et al. [32] proposed the Isogeometric Analysis (IGA) to provide a link between CAD and CAE-FEA by applying the same higher-order basis functions commonly B-splines and NURBS functions for both design and analysis. A positive side effect of IGA basis functions is their higher-order continuity which has been exploited for thin shell analysis [32–34], phase field models [35–37] and gradient models [31]. In this manuscript, both second-order and fourth-order implicit stress-based gradient-enhanced formulations are constructed in the context of the isogeometric finite element method.

The paper is organized as follows: In Section 2, the continuum damage theory is briefly recalled and the stress-based gradient enhanced model is introduced. Section 3 focuses on the isogeometric finite element approach for the proposed model. Two numerical examples are described in Section 4, where the performance of the proposed model in terms of mesh dependence, damage initiation and propagation are investigated.

2. Continuum damage theory

2.1. Constitutive equations of isotropic damage models

We assume small strain theory. Damage is considered to be isotropic and a scalar damage variable ω is introduced to describe the damage process. The stress–strain relation is given by

$$\boldsymbol{\sigma} = (1-\omega)\mathbb{C} : \boldsymbol{\epsilon} = \mathbb{C}^{eff} : \boldsymbol{\epsilon}, \tag{1}$$

where $\mathbb{C}$ is the fourth-order elasticity tensor and $\mathbb{C}^{eff} = (1-\omega)\mathbb{C}$ is the effective elastic tensor and ω is the damage parameter ($\omega = 0$ for undamaged state and $\omega = 1$ for completely damaged state). The damage parameter is expressed in terms of a state variable and can only monotonically increase. The Kuhn–Tucker relations are given by

$$\begin{cases} f \le 0. \\ \dot{\kappa} \ge 0, \\ f\dot{\kappa} = 0, \end{cases} \tag{2}$$

where the loading function f depends on the equivalent strain ϵ_{eq} and the history parameter κ:

$$f = f(\epsilon_{eq}, \kappa). \tag{3}$$