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Nonlinear frequency response based adaptive vibration controller design for a class of nonlinear systems

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ABSTRACT

Frequency response functions (FRF) are often used in the vibration controller design problems of mechanical systems. Unlike linear systems, the FRF derivation for nonlinear systems is not trivial due to their complex behaviors. To address this issue, the convergence property of nonlinear systems can be studied using convergence analysis. For a class of time-invariant nonlinear systems termed as convergent systems, the nonlinear FRF can be obtained. The present paper proposes a nonlinear FRF based adaptive vibration controller design for a mechanical system with cubic damping nonlinearity and a satellite system. Here the controller gains are tuned such that a desired closed-loop frequency response for a band of harmonic excitations is achieved. Unlike the system with cubic damping, the satellite system is not convergent, therefore an additional controller is utilized to achieve the convergence property. Finally, numerical examples are provided to illustrate the effectiveness of the proposed controller.

1. Introduction

Mechanical vibrations are present in countless real-life situations, where the mechanical systems exhibit oscillations when subjected to certain excitations. Most often, these vibration phenomena are highly undesirable, which may even cause damage to the system itself. The vibration control is concerned with the prediction and controlling of any undesired oscillations present in both linear and nonlinear systems. There exist several time-domain based vibration controller design techniques [1]. However, these methods lack in describing how a closed-loop system respond to the disturbances at different amplitudes and frequencies. Since the vibration is characterized by its frequency, amplitude, and phase, it is important to study the frequency response of these systems [2].

Frequency-domain techniques for linear time-invariant (LTI) systems have led to significant progress in analysis, modeling, and controller design [3–5]. Unlike LTI systems, many practical mechanical systems possess nonlinear dynamic behavior caused by material nonlinearity, geometric nonlinearity, damping dissipation, boundary conditions, and so on [6]. These nonlinearities can result in some complex phenomenon like jumping, chaos, secondary resonance, and bifurcation, which lead to nonlinear vibration problems [7]. The traditional linear frequency-domain techniques are insufficient to describe these complex behaviors of the nonlinear system adequately.

Due to the popularity and importance of the frequency-domain techniques, a number of interesting nonlinear frequency response techniques have been proposed and analyzed in the past few decades. These include generalized frequency

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response function (GFRF), output frequency response function (OFRF), describing function (DF), etc. Nonlinear FRF such as the GFRF [8] is limited to second order due to its multi-dimensional characteristics. An extension of the GFRF termed as OFRF was proposed in [9,10], which represents the relation between the system parameters and frequency response using Volterra series. DF method such as higher-order sinusoidal input describing function (HOSIDF) was developed to describe the FRF of a linearized system to a sinusoidal input signal [11]. But all the aforementioned techniques lead to an approximate nonlinear FRF.

Obviously, the existence of a unique steady-state solution must be ensured in the FRF derivation. The nonlinear phenomena such as dependency upon the initial conditions and existence of multiple steady-state solution to the same input, can bring some difficulties on the development of a nonlinear FRF [6,7]. In [12,13], it has been shown that for a certain class of nonlinear time-invariant systems termed as convergent systems, a periodic input will result in a unique steady-state output with the same period of the input. Since this property agrees with the LTI case, an FRF can be obtained simply by finding the ratio of the amplitude at the steady-state output and input amplitude [13]. Unlike the GFRF, OFRF, and DF, this approach provides an exact nonlinear FRF.

The main objective of this paper is to introduce the potential of convergence analysis and nonlinear FRF in vibration control problems. This paper considers a nonlinear frequency response based adaptive vibration controller design for a class of nonlinear time-invariant mechanical systems. In order to derive the nonlinear FRF, first the existence of a unique steady-state have been established theoretically. Later, the FRF is obtained numerically for a band of harmonic excitations. In terms of control, the controller gains are adapted based on the FRF of the system, such that a satisfactory performance over the excitation band of interest is obtained. The control scheme developed here is applied to the vibration control problem of a mechanical system with cubic damping nonlinearity and a satellite system, and the corresponding dynamic responses of both the controlled and uncontrolled cases are numerically evaluated.

2. Preliminaries

2.1. Contraction theory

For a dynamic system to be satisfactory, it is necessary to analyze its stability. In general, a system's stability is analyzed by examining whether the equilibrium points so determined are stable. The contraction analysis is inspired by the theory of fluid mechanics and differential geometry. Unlike the Lyapunov stability theorem which defines the stability with respect to the equilibrium points, the contraction in a contracting system implies that the system trajectories with different initial conditions will converge to each other [12].

Definition 2.1. Let a dynamical nonlinear system be described by the differential equation

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \tag{1}$$

where $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$ is a smooth nonlinear function, $\mathbf{x} \in \mathbb{R}^n$ is the state vector, and $t \in \mathbb{R}$. For the above system, a contraction region Ω_x is defined where the system's Jacobian matrix $\mathbf{J} = \partial\mathbf{f}/\partial\mathbf{x}$ is uniformly negative-definite [12].

Uniform negative-definiteness of $\mathbf{J}$ implies

$$\mathbf{J}_{\text{sym}} \leqslant \lambda_{\max}\mathbf{I}_n \leqslant -\beta\mathbf{I}_n < 0, \quad \forall\mathbf{x} \in \Omega_x \subset \mathbb{R}^n, \ \forall t \in \mathbb{R} \tag{2}$$

where $\mathbf{J}_{\text{sym}} = \frac{\mathbf{J}+\mathbf{J}^{\mathrm{T}}}{2}$, $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{J}_{\text{sym}}$, $\beta > 0$, and $\mathbf{I}$ is the identity matrix.

The contraction of the system (1) can also be verified by performing a coordinate transformation on $\mathbf{J}$ [12]. The resulting generalized Jacobian $\mathbf{J}_{\text{gen}}$ is defined as

$$\mathbf{J}_{\text{gen}} = \left(\dot{\mathbf{T}} + \mathbf{TJ}\right)\mathbf{T}^{-1} \tag{3}$$

where $\mathbf{T}(\mathbf{x}, t)$ is a uniformly invertible square matrix. If $\mathbf{J}_{\text{gen}}$ is uniformly negative-definite, then the transformed system (3) is contracting, which implies that all the solutions of the original system (1) converge exponentially to a single trajectory, irrespective of the initial conditions. If $\mathbf{J}_{\text{gen}}$ is negative semi-definite, under some mild conditions similar in Barbalat's lemma, the system is semi-contracting, which implies that the solutions converge each other asymptotically. The contraction or semi-contraction is achieved globally when $\Omega_x = \mathbb{R}^n$.

The contraction analysis is quite a useful tool for designing observers [14,15] and controllers [16] and for the synchronization problems [17,18]. If the observer equation contains a unique solution which is contracting, then the observer will estimate the right states. In the case of a control design, the controller will force the system trajectories to a desired performance from any initial condition. In synchronization problems, multiple systems are forced to follow a common desired trajectory.

For the time-invariant case, if the contracting system is driven by a periodic input, then the corresponding system solution tends exponentially to a periodic solution with the same period [12]. The property described here can be extended to the frequency response techniques, which will be discussed in terms of convergence analysis in the following subsection.