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Efficient reliability analysis of structures with the rotational quasi-symmetric point- and the maximum entropy methods

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ABSTRACT

This paper presents a new method for efficient structural reliability analysis. In this method, a rotational quasi-symmetric point method (RQ-SPM) is proposed for evaluating the fractional moments of the performance function. Then, the derivation of the performance function's probability density function (PDF) is carried out based on the maximum entropy method in which constraints are specified in terms of fractional moments. In this regard, the probability of failure can be obtained by a simple integral over the performance function's PDF. Six examples, including a finite element-based reliability analysis and a dynamic system with strong nonlinearity, are used to illustrate the efficacy of the proposed method. All the computed results are compared with those by Monte Carlo simulation (MCS). It is found that the proposed method can provide very accurate results with low computational effort.

1. Introduction

A fundamental problem in reliability analysis of structures is the determination of the probability of failure p_f [1], which is defined by

$$p_f = Pr[G(\mathbf{Z}) \leqslant 0] = \int_{G(\mathbf{Z})\leqslant 0} f_{\mathbf{Z}}(\mathbf{z})d\mathbf{z} \tag{1}$$

where Pr stands for probability, $G(\cdot)$ is the explicit or implicit limit state function or the performance function, $\mathbf{Z} = \{Z_1, Z_2, \ldots, Z_d\}^T$ is the random vector consisting of d basic random variables involved in both structural parameters and loads, and $f_{\mathbf{Z}}(\mathbf{z})$ denotes the joint probability density function (PDF) of $\mathbf{Z}$.

Despite the simplicity of the formulation of the problem, the exact solution of Eq. (1) is difficult since the explicit expressions of limit state functions, in general, are not available for realistic engineering problems, particularly when nonlinear behavior needs to be considered [2]. Difficulty in computing this probability has led to the development of various approximation techniques, among which three kinds of methods are usually adopted. The first kind is the first- or the second-order reliability method (FORM or SORM) [3]; those methods were considered to be the most popular for reliability analysis in the

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past decades. However, in the reliability evaluation of nonlinear structures using direct finite element formulations, the derivatives of limit state functions with respect to basic random variables, required for FORM/SORM, may be not readily available [4]. On the other hand, the most widely adopted methods to determine the probability of failure are the Monte Carlo simulation (MCS) and its variants, which are grouped under the second kind. The MCS in its crude form may require excessive computational efforts in evaluating problems of small failure probability or in problems involving a large number of costly finite element analysis. Therefore, in the past decades, some remarkable improvements on the efficiency of MCS have been made, which could be called the advanced MCS. Representative contributions include subset simulation [5], asymptotic sampling [6] and line sampling [7], etc. The third is the response surface methodology [8], which builds a surrogate model for the target limit state function, defined in a simple and explicit mathematical form for reliability analysis of linear and nonlinear structures [9,10]. Response surface models can be built to find the design point with a much reduced computational cost [11], where FORM or MCS can be applied for reliability analysis with high efficiency. Here it is convenient to explicitly mention neural networks [12,13], support vector machines [9,14,15] and Gaussian processes (kriging) [16] as representative methodologies of response surface models. Extensive developments have been studied in the aforementioned methods. Nevertheless, other efficient reliability methods still attract interest in the structural reliability community.

Another route for obtaining structural reliability or probability of failure is directly utilizing the probability density function (PDF) of the performance function. This subject could be classified into two categories. The first is the probability density evolution method (PDEM) [2,17–21], which straightforwardly derives the PDF from a partial differential equation. The second is the method of moments [22], which approximates the distribution of a random variable using its moments of finite orders. Specifically, in the present paper, the method of moments is of interest; this method requires the computation of moments of a multivariate function involving multi-dimensional integrals and therefore numerical integration methods, which can keep the tradeoff between efficiency and accuracy, are preferred. Among the numerical methods for multi-dimensional integration, the univariate dimension reduction method (UDRM) [23], the high dimensional model representation method (HDMRM) [24,25] and the quasi-symmetric point method (Q-SPM) [26,27] might be the best choices. The virtue of these methods is that they need considerably fewer response function evaluations in comparison to other simulation based methods. The UDRM decomposes a multi-dimensional integral into several one-dimensional integrals; it has been used to capture the moments information for reliability analysis [1,28]. However, this method may be not adequate for a system with a large number of random variables or strong nonlinearity due to the high-dimensional integrals retained in the residue error of this method [29]. On the other hand, since the higher order terms in HDMRM are usually negligible, then the first-order and the second-order HDMRM are mostly adopted. In fact, the first-order HDMRM is equivalent to the UDRM while the second-order HDMRM might be called the bivariate dimension reduction method (BDRM). Although the second-order HDMRM provides more accurate results for general nonlinear structures, the computational efforts may significantly increase compared with that of UDRM [30]. The quasi-symmetric point method (Q-SPM), which is established based on the invariant theory and orthogonal arrays with a 5th degree of algebraic accuracy for numerical integration, is applicable to linear/nonlinear systems with multiple random parameters. The number of deterministic analysis required in Q-SPM is quite close to that of UDRM, which indicates the high efficiency of Q-SPM. In a recent implementation of this method [31], the Q-SPM was found to be not accurate enough for stochastic dynamic response analysis of structures. An improvement of Q-SPM for the computation of moments is necessary. Besides, the adaptability of this method for structural reliability analysis has not been explored yet.

The objective of the present paper is to develop an efficient method based on the rotational quasi-symmetric point method (RQ-SPM) and the maximum entropy method (MEM) for structural reliability analysis. The paper is arranged as follows. Section 2 devotes to introducing the fundamentals of the quasi-symmetric point method. In Section 3, a new rotational quasi-symmetric point method is proposed for calculation of the fractional moments of the performance function's probability density function (PDF). Determination of the PDF of the performance function by the maximum entropy method in such a manner that its fractional moments are used as constraints is conducted in Section 4. In Section 5, integration of the performance function's PDF over the failure domain gives the probability of failure. Numerical examples are investigated in Section 6 to verify the proposed method. Concluding remarks and the problems to be further studied are included in the final section.

2. Fundamentals of the quasi-symmetric point method

The quasi-symmetric point method (Q-SPM) was proposed by Victoir [26] for the purpose of Gaussian weighted multi-dimensional numerical integration. This method has also been applied to stochastic dynamic response analysis of structures in Ref. [27]. Brief basics of the Q-SPM are given in this section.

Let us define the l-th moment of the performance function $G(\mathbf{Z})$ as

$$m_l = E[G^l(\mathbf{Z})] = \int_{\Omega_{\mathbf{Z}}} G^l(\mathbf{Z}) f_{\mathbf{Z}}(\mathbf{z}) d\mathbf{z} \tag{2}$$

where l is a natural number, $\Omega_{\mathbf{Z}}$ is the distribution domain of $\mathbf{Z}$.

It is well known that non-normal random variables might be transformed into standard normally distributed random variables by some specific transformation, e.g. Rosenblatt transformation [32]. In consequence, Eq. (2) can be evaluated by

$$m_l = \int_{\Omega_{\mathbf{X}}} G^l(R^{-1}[\mathbf{X}]) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \approx \sum_{k=1}^{N} a_k G^l(R^{-1}[\mathbf{x}_k]) \tag{3}$$