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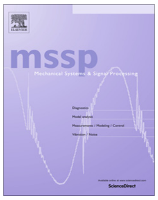

Modal parameter identification by an iterative approach and by the state space model

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ABSTRACT

The problem of estimating a spectral representation of exponentially decaying signals from a set of sampled data is of considerable interest in several applications such as in vibration analysis of mechanical systems. In this paper we present a nonparametric and a parametric method for modal parameter identification of vibrating systems when only output data is available. The nonparametric method uses an iterative adaptive algorithm based in the formation of a two dimensional grid mesh, both in frequency and damping domains. We formulate the identification problem as an optimization problem where the signal energy is obtained from each frequency grid point and damping grid point. The modal parameters are then obtained by minimizing the signal energy from all grid points other than the grid point which contains the modal parameters of the system. The parametric approach uses the state space model and properties of the controllability matrix to obtain the state transition matrix which contains all modal information. We discuss and illustrate the benefits of the proposed algorithms using a numerical and two experimental tests and we conclude that the nonparametric approach is very time consuming when a large number of samples is considered and does not outperform the parametric approach.

1. Introduction

In many applications the mechanical structure under consideration is excited by a brief input signal, called an impulse. This is particularly so in mechanical vibration analysis test situations, where the test system is excited by a force acting over a very short duration time: for example, the system is struck by a hammer. Other common types of vibration testing services are conducted using a step force which is constant through a time frame. The corresponding response of the system to an impulse or a step force is a transient response, since steady-state oscillations are not produced and oscillations of the system reduce to zero after a finite time. Consequently, transient responses are short time events whose time behavior cannot be predicted and are totally varying in nature, both in time and frequency. Two fundamental parameters to identify from the transient response of a vibrating system are the eigenfrequencies and the damping coefficients and we are faced to a problem of spectral analysis. The problem of estimating the spectral parameters of damped signals has attracted significant attention during recent decades as such signals arise naturally in many engineering domains. We can mention the vibration monitoring [1,2], where the spectral content of measured signals gives information on the wear of mechanical parts under study, the magnetic resonance spectroscopy [3] to obtain further information about cellular activity, in speech analysis [4] for speech synthesis and speech recognition, in geophysical seismology [5] to predict subsurface geologic structure, in sonar and in radar [6] to estimate the locations and waveforms of acoustical or electromagnetic sources. Nonparametric and

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parametric spectral estimators are usually used [7]. A nonparametric estimator attempts to compute the spectral content of a signal without using any a priori information about it: there is no assumption about how the data is generated. The most common nonparametric estimator is the traditional periodogram which basically reduces to the computation of a Discrete Fourier Transform and gives a power spectral density estimate of the signal. Bartlett, Daniell and Welch [7] have proposed refined periodograms by smoothing or averaging the traditional periodogram obtained from segments which are windowed. Recently, Gudmundson [8,9] has implemented an iterative adaptive approach for radiofrequency spectroscopy in the case of irregularly sampled data. The algorithm uses data from nuclear quadrupole resonance experiments and is employed to determine ammonium nitrate and cyclonite in a liquid.

A parametric or model-based estimator assumes that the structure of the signal is known: the signal satisfies a generating model with known functional form. Parametric methods are used to estimate the parameters in the assumed model and the most common of the parametric estimation technique is the auto-regressive moving average (ARMA) modeling of the signal [10]. The modal parameters are obtained from eigendecomposition of the companion matrix which is formed with the AR coefficients. The ARMA approach, unlike the nonparametric approaches, requires the selection of the order, or the structure, of the model. The selection of model order is a problem always under investigation. Other parametric methods using subspace algorithms and derived from the state space model have been developed and are always part of an intense research [11–15]. Note that none of these parametric methods is able to cope with very low snapshot numbers or with a severe measurement noise. The problem of measurement noise has been investigated by Juang and Papa in [16] where the errors characteristics on modal parameters due to noise have been analyzed by numerical simulations using the Monte Carlo technique and the eigensystem realization algorithm. More recently Dorvash and Pakzad [17] have presented the influence of measurement noise on modal parameter identification using experimental tests and the eigensystem realization algorithm. It is shown that the deviation of the modal parameters obtained by the low-noise sensors is generally less than the deviation of the modal parameters identified by the sensors with higher noise level. The authors concluded that the attenuation of this deviation can be performed by increasing the model order of the system and through the use of a stabilization diagram.

To estimate the eigenfrequencies and damping coefficients of a vibrating system from output data only we propose a nonparametric iterative adaptive algorithm and a parametric approach based on a subspace algorithm. The iterative adaptive technique does not require the specification of any user parameters. Furthermore, no assumptions have to be made on the sampling scheme and the spectrum can be estimated from regularly or even irregularly sampled data [3]. To our knowledge it is the first time that the iterative adaptive method is applied to modal parameter identification of mechanical systems and constitutes a novelty. Furthermore, this algorithm uses a two-dimensional grid mesh, both in frequency and damping domain and a strategy to choose the grid mesh is presented.

The proposed subspace algorithm, which is derived from the state space model, is based on shifted properties of the controllability matrix. The matrices operations that yield from the controllability matrix to the state transition matrix are a peculiarity of the paper. To illustrate the effectiveness of the proposed algorithms, we examine the results on simulated data from exponentially damped sinusoids corrupted by white noise, on experimental measurements from the displacement of a micro electro mechanical system (MEMS) constituted of a perforated microplate and on measurements from the vibrations of a line cable excited through an impact hammer. Note that to our knowledge it is the first time that the proposed subspace algorithm is used to identify the modal parameters of a MEMS and one of the goals of this work is to compare the iterative adaptive algorithm and the subspace algorithm using numerical and experimental tests.

The paper is outlined as follows. In the next section we present the data model and derive a nonparametric approach based on the iterative adaptive algorithm for modal parameter identification. In Section 3 we propose a parametric approach derived from the state space model with a subspace algorithm. The performances of the proposed methods are studied in Section 4 using both simulated and measured data obtained from the movement of a perforated microplate and from the vibrations of a line cable. This paper is briefly concluded in Section 5.

2. The iterative adaptive algorithm in mechanical vibrations

2.1. Response of a MDOF system

The linear MDOF system is governed by the general equation

$$\mathbf{M}\,\ddot{\mathbf{Z}}(t) + \mathbf{C}\dot{\mathbf{Z}}(t) + \mathbf{K}\,\mathbf{Z}(t) = \mathbf{F}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are mass, damping and stiffness matrices respectively, and $\mathbf{F}(t)$ is the excitation vector. The response $\mathbf{Z}(t)$ of the system can be obtained using well known modal analysis or a direct forced response method. Instead of the MDOF system given by (1), N uncoupled equations similar to a SDOF system can be obtained

$$m_i\,\ddot{z}(t) + c_i\dot{z}(t) + k_i\,z\,(t) = f_i(t) \tag{2}$$

for $i = 1, 2 \ldots, N$.

The impulse response of this MDOF system is obtained as the superposition of the N relevant modes

$$h(t) = \sum_{i=1}^{N} A_i e^{(-\xi_i\omega_{ni}t)}\cos(\omega_{di}t + \varphi_i) \tag{3}$$