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Comput. Methods Appl. Mech. Engrg. 304 (2016) 619–655

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Phase field modeling of fracture in multi-physics problems. Part III. Crack driving forces in hydro-poro-elasticity and hydraulic fracturing of fluid-saturated porous media

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Received 13 January 2015; received in revised form 11 September 2015; accepted 13 September 2015
Available online 19 October 2015

Abstract

The prediction of fluid- and moisture-driven crack propagation in deforming porous media has achieved increasing interest in recent years, in particular with regard to the modeling of *hydraulic fracturing*, the so-called "*fracking*". Here, the challenge is to link at least three modeling ingredients for (i) the behavior of the solid skeleton and fluid bulk phases and their interaction, (ii) the crack propagation on not a priori known paths and (iii) the extra fluid flow within developed cracks. To this end, a macroscopic framework is proposed for a continuum phase field modeling of fracture in porous media. It provides a rigorous geometric approach to a *diffusive* crack modeling based on the introduction of a constitutive *balance equation for a regularized crack surface* and its modular linkage to a Darcy–Biot-type bulk response of hydro-poro-elasticity. The approach overcomes difficulties associated with the computational realization of sharp crack discontinuities, in particular when it comes to complex crack topologies including branching. A *modular concept* is outlined for linking of the diffusive crack modeling with the hydro-poro-elastic response of the porous bulk material. This includes a generalization of crack driving forces from energetic definitions towards threshold-based criteria in terms of the *effective stress* related to the solid skeleton of a fluid-saturated porous medium. Furthermore, a Poiseuille-type constitutive continuum modeling of the extra fluid flow in developed cracks is suggested based on a deformation-dependent permeability, that is scaled by a characteristic length. This proposed modular model structure is exploited in the numerical implementation by constructing a robust finite element method, based on an algorithmic decoupling of updates for the crack phase field and the state variables of the hydro-poro-elastic bulk response. We demonstrate the performance of the phase field formulation of fracture for a spectrum of model problems of hydraulic fracture. A slight modification of the framework allows the simulation of *drying-caused crack patterns* in partially saturated capillar-porous media.

Keywords: Fracture; Porous media; Hydraulic fracturing; Drying processes; Phase field modeling; Coupled problems

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http://dx.doi.org/10.1016/j.cma.2015.09.021
0045-7825/

1. Introduction

1.1. Fluid-driven hydraulic fracture of porous media

The computational modeling of fluid- and moisture-driven crack propagation in porous media has achieved increasing research activities in recent years. Predictive simulations are needed for numerous practical applications in geotechnical, environmental and petroleum engineering, material science, biomechanics and medical science. Typical examples cover the propagation of magma-driven dikes, fault activation in mining, drying-induced fracture in porous materials and failure of fluid-saturated biological structures. Currently, major research is devoted to the computational modeling of *hydraulic fracturing*, the so-called "fracking", due to the growing interest of the petroleum industry. The works Boone and Ingraffea [1], Rubin [2], Zhang et al. [3], Adachi et al. [4], Bažant et al. [5] and Simoni and Schrefler [6] provide an introduction to the subject. Following conceptually our works [7,8], we outline in this article a new phase field approach to the modeling of fluid-driven fracture in porous media.

The challenge in simulation of complex crack patterns in porous media is a robust modeling of (i) the behavior of the solid skeleton and fluid bulk phases and their interaction, (ii) the crack propagation on not a priori known paths and (iii) the extra fluid flow within developed cracks. The process of fluid flow in a deforming porous medium was first considered in Terzaghi [9], and extended by Biot [10,11] and Rice and Clearly [12] to a general *macroscopic continuum theory* that includes Darcy's law of fluid transport. Biot's theory is based on an a priori existence of a macroscopic total stress that decomposes into intergranular and fluid actions. In contrast, microscopically based *mixture* or *multi-phase theories* represent the porous medium by spatially superposed interacting media with separate balance equations and constitutive functions for the solid skeleton and the fluid phase, and couple them by appropriate constitutive assumptions. We refer to Bowen [13], Bedford and Drumheller [14], Truesdell [15], Coussy [16], de Boer [17] and Ehlers [18]. We base the subsequent developments of phase field fracture on a geometrically nonlinear framework of hydro-poro-elasticity at finite strains in line with the work of Biot [11], see also Detournay and Cheng [19] and Coussy et al. [20] for a justification of the macroscopic approach. A conceptual extension of the fracture model outlined below to the above mentioned mixture theories is straightforward.

The modeling of fracture in porous media, especially hydraulically driven fracture, is very complex because numerous effects need to be considered. First, the above mentioned coupled problem of fluid transport in a deformable porous medium has to be linked with a method describing the initiation and propagation of cracks. Furthermore, the fluid flow along the cracks needs to be modeled. For the latter, the classical *Poiseuille law* for laminar flow of an incompressible viscous fluid is often applied, see e.g. Adler et al. [21] for a derivation. It can be combined with a separate transport equation for the fluid along a crack, yielding a statement for the evolution of the crack opening, see Boone and Ingraffea [1], Rubin [2] and Adachi et al. [4]. Alternatively, the permeability in Darcy's law can be made dependent on the crack opening, see Schrefler et al. [22]. Analytical solutions for fracture scenarios in poroelastic solids are outlined by Rice and Clearly [12], Huang and Russell [23,24], Zhang et al. [3], Savitski and Detournay [25] and Garagash and Detournay [26]. One of the first attempt to model hydraulic fracture numerically is documented in Boone and Ingraffea [1], who combined a finite element method for the poro-elastic bulk response with a finite difference method for the fluid flow along the crack. Fracture was modeled by a cohesive zone model for *predetermined crack paths* discretized by *interface elements*. Adaptive meshing strategies with cohesive interface elements for *not a priori known crack paths* are considered in Schrefler et al. [22] and Secchi and Schrefler [27,28]. *Strong discontinuity methods* with crack discontinuities embedded into finite elements are treated in the context of localization analyses by Larsson et al. [29], Steinmann [30], Callari and Armero [31] and Callari et al. [32]. An *extended finite element method* for the description of fracture in poro-elastic media is used by de Borst et al. [33] in a geometrically linear setting and Irzal et al. [34] in a nonlinear setting at finite strains. Kraaijeveld et al. [35] applied the method to saturated porous media with coupled ion-diffusion. Further applications of this approach are documented in Réthoré et al. [36] and Mohammadnejad and Khoei [37]. Approaches to hydraulic fracturing beyond finite element methods are Gordeliy and Detournay [38] and Grassl et al. [39].

1.2. A phase field approach to hydraulic fracture

However, the above reviewed computational modeling of *sharp* crack discontinuities suffer in situations with complex crack topologies including branching. This can be overcome by recently developed phase field approaches